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AD A105068

# Reliability and Service Life Concepts
# for Sonar Transducer Applications

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September 17, 1981

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| REPORT DOCUMENTATION PAGE | READ INSTRUCTIONS BEFORE COMPLETING FORM |
|---|---|

| 1. REPORT NUMBER | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER |
|---|---|---|
| NRL Memorandum Report 4558 | AD-A105068 | |

| 4. TITLE (and Subtitle) | 5. TYPE OF REPORT & PERIOD COVERED |
|---|---|
| RELIABILITY AND SERVICE LIFE CONCEPTS FOR SONAR TRANSDUCER APPLICATIONS | Interim report on a continuing problem. |
| | 6. PERFORMING ORG. REPORT NUMBER |

| 7. AUTHOR(s) | 8. CONTRACT OR GRANT NUMBER(s) |
|---|---|
| R. L. Smith | N00173-81-M-T141 |

| 9. PERFORMING ORGANIZATION NAME AND ADDRESS | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
|---|---|
| Texas Research Institute, Inc. 5902 West Bee Caves Road Austin, Texas 78746 | 64503N; S0219, AS; 0584-7 |

| 11. CONTROLLING OFFICE NAME AND ADDRESS | 12. REPORT DATE |
|---|---|
| Underwater Sound Reference Detachment Naval Research Laboratory P.O. Box 8337, Orlando, Florida 32856 | September 17, 1981 |
| | 13. NUMBER OF PAGES |
| | 114 |

| 14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) | 15. SECURITY CLASS. (of this report) |
|---|---|
| Naval Sea Systems Command Washington, DC 20362 | UNCLASSIFIED |
| | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

This report was compiled 30 November 1980 by TRI under contract funded by the Sonar Transducer Reliability Improvement Program.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Reliability
Sonar transducers
Service lifetime

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In recent years, the Navy has begun to include reliability requirements in procurement specifications for wet-end sonar hardware. For the most part, the requested reliability evaluations have focused on exponential modeling and the use of handbook methods originally developed primarily for electronics systems. In this report, we examine the relevance of this and other approaches. Reliability concepts are reviewed without

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S/N 0102-014-6601

20   ABSTRACT (Continued)

restricting their scope to the description of a single class of operating behavior. The discussion begins with the indoctrination of the mathematical functions most commonly used in reliability descriptions. Modeling of the constant, increasing, and decreasing hazard rate situations is discussed. Reliability and service-life concepts are compared and contrasted from the prediction viewpoint. The task of demonstrating reliability in as-built equipment is also dealt with. The report calls attention to some of the special problems such as limited production and long intended life associated with evaluating sonar equipment reliability. It concludes with several recommendations directed toward the systematic improvement of sonar hardware.

## TABLE OF CONTENTS

iii

iv

## LIST OF TABLES

## LIST OF ILLUSTRATIONS

# RELIABILITY AND SERVICE LIFE CONCEPTS FOR SONAR TRANSDUCER APPLICATIONS

## 1.0 INTRODUCTION AND SUMMARY

In recent years, the Navy has begun to include reliability requirements in procurement specifications for wet-end sonar hardware. For the most part the requested reliability evaluations have focused on exponential modeling and the use of handbook methods originally developed primarily for electronics systems. In this report we examine the relevance of this and other approaches. Reliability concepts are reviewed without restricting their scope to the description of a single class of operating behavior. The discussion begins with the introduction of the mathematical functions most commonly used in reliability descriptions. Modeling of the constant, increasing, and decreasing hazard rate situations is discussed.

Reliability problems tend to have strongly statistical aspects. This leads us to deal with probability ideas and the properties of distributions. Reliability and service life concepts are compared and contrasted from the prediction viewpoint. The task of demonstrating reliability in as-built equipment is also dealt with. The report calls attention to some of the special problems such as limited production and long intended life associated with evaluating sonar equipment reliability. It concludes with several recommendations directed toward the systematic improvement of sonar hardware.

Most of the material presented here was developed in the open periodical literature and now has been refined and cataloged in standard reliability texts. However, as a distinct autonomous discipline, reliability studies are only about 35 years old. There seems to be an important fragmentation between advocates of handbook-style prediction and probabilistic design practicioners. This author views the two approaches as complementary--each with advantages and limitations. An effort is made in this report to provide the background to permit progress on sonar problems from both points of view. Of necessity the scope of this must be limited. Hopefully, however, a basis for more specific and detailed studies is established.

An attempt has been made to present the material in sufficient detail and rigorously enough to serve the technical needs of managers of sonar upgrading and procurement programs and other workers in the field. Obviously to realize the economic benefits that usually accompany well-structured reliability efforts, the importance of this kind of pursuit cannot be overemphasized.

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1

## 2.0 STANDARD MODELING CONCEPTS

Discussions of reliability topics commonly begin with a definition of reliability. There is some variety among reliability definitions but a representative example might be: Reliability is the probability that an equipment will satisfactorily perform its intended function for a specified time when operated in the manner and for the purpose intended. This statement conveys a general impression of the reliability concept but is incomplete. It needs to be supported by specifications of the nature of the probability statement, what constitutes satisfactory equipment performance, mission duration, environmental exposures, proper use, etc. It is possible to extend the reliability definition to include the important statements of qualification. Clarity often suffers when this is done. In contrast the statement can be streamlined to simply, reliability is the probability of success. Again communication on the subject involves clarifying a number of related circumstances.

Measuring reliability involves quantifying a probability statement. Thus single unit reliability is not directly observable but must be inferred from other information relating to failed units within a population. The important relationships are developed and cataloged in Section 2.1. Sections 2.2, 2.3, and 2.4 deal with specific examples of situations exhibiting constant, increasing, and decreasing hazard rates.

## 2.1 Some Reliability Functions and Interrelationships

Several important reliability functions and relationships are displayed in Table I. The first 6 line entries are functions commonly encountered in reliability theory. Actually the MTBF is not a function but rather a statistic (measuring the central tendency of $f(t)$, the time-to-failure probability density function). Probably the MTBF acquires its status because its specification in a one-parameter model (such as the exponential case) completely characterizes the description. The remaining reliability functions all depend on time $t$ here taken to represent operating time or the age of the component/equipment/system since manufacture or installation.

The second portion of Table I gives a number of relationships connecting the various reliability functions. Unreliability is defined via Eq. (2) as the cumulative of the time-to-failure distribution function. Unreliability and reliability are complementary functions via Eq. (6). Equations (2) and (6) imply Eq. (1a). Equation (1b) is derived in Appendix A. Differentiation of Eq. (2) under the integral sign leads to Eq. (3a) while use of Eq. (6) further implies Eq. (3c). Differentiation of Eq. (1b) implies Eq. (4a) and its equivalent Eq. (4b). Use of Eq. (3c) in Eq. (4a) leads to Eq. (3b). Equation (5a) defines MTBF and parts integration implies Eq. (5b) as an equivalent statement. Equation (7) defines conditional reliability as a function of age $t$ at the start of a mission and mission duration T.

The reliability functions of Table I are so richly interconnected that specification of any one of the functions $R(t)$, $U(t)$, $f(t)$, or $\lambda(t)$ implies all the other quantities of interest including MTBF and $R(t,T)$. In contrast, specifying MTBF alone places a single constraint on reliability modeling parameters and implies a complete description only in the case of a single-parameter model.

2

## 2.2      Random Hazard Case--Exponential Reliability

In order to better visualize the common forms of reliability modeling, let us graphically relate relia ility and expected failure times to the underlying hazard function $\lambda(t)$. Many systems experience a fairly stable minimum hazard rate only after a period of operation that separates congenitally inferior units from the rest of the population of similar items. The weak units are referred to as early failures and the time domain of their occurrence is termed the infant mortality region. A period of stable hazard rate is referred to as the random hazard region. Typically as damage to the system accumulates the hazard rate increases rapidly, signaling entry into the wearout phase. Hazard functions for electronic and mechanical components are sketched in Fig. 1. Electronic components tend to exhibit a pronounced region where the hazard rate is constant as shown in Fig. 1a (the celebrated "bathtub" curve of reliability studies). Wearout as suggested in Fig. 1b, tends to be more prominent in mechanical systems. In this section of the report, we consider the reliability implications of a constant hazard rate. Early and wearout reliablity are dealt with in the sections to follow.

The region of stable hazard rate is referred to as the random hazard region because a failure is equally as likely to occur in any one time interval as in any other such interval of equal duration. Applying Eq. (1b) to this situation ($\lambda$ = const.) yields for the random hazard reliability

$$R(t) = e^{-\lambda t}. \tag{8}$$

Thus constant hazard rate implies exponential reliability. The time-to-failure probability density function is obtained by using Eq. (8) in Eqs. (3b) or (3c). Thus,

$$f(t) = \lambda e^{-\lambda t}. \tag{9}$$

The function $f(t)$ is itself an exponential (scaled as $\lambda$) function. Equations (8) and (9) follow from a constant hazard rate $\lambda$. For completeness we include the letter statement explicitly as

$$\lambda(t) = \lambda. \tag{10}$$

Equations (8), (9), and (10) characterize what is usually referred to as the random hazard or exponential reliability situation. Representative random hazard reliability functions are plotted in Fig. 2. In practice care must be exercised to make sure the random hazard description is used appropriately. This may mean eliminating early failures via burn-in techniques or avoiding the wearout region by limiting the time domain wherein Eqs. (8) through (10) are used.


## 2.3      Normally Distributed Times to Failure--Wearout

In the previous section it was convenient to use constancy of the hazard function as a point of departure. This corresponded to a static reliability situation in which the vulnerability of the system or component of interest under applied stresses did not change with time. This is a proper description of many real-life reliability problems. There are also numerous situations wherein the perfor-

3

mance attributes of the item of interest degrade with time. A wide variety of wearout phenomena such as fatigue, corrosion, sputtering, abrasion, diffusion, etc., operate to populate this category. In general, wearout is characterized by the systematic loss of system or component performance due to material property changes or outright loss of working substance. The changes may be induced by applied thermal or mechanical loads--sputtering, diffusion, and fatigue crack growth are examples. Degradation processes such as corrosion and diffusion may also proceed independently of applied load. Synergistic effects such as stress corrosion or corrosion fatigue also occur.

Loss of function to wearout translates into an increasing vunerability to catastrophic failure in service under normal application of stress. Thus, the hazard function (probability per unit time of experiencing a failure) is an increasing function as damage to the system or component of interest accumulates. The variety of wearout processes and variability of loading situations often render directly characterizing the shape of the increasing hazard function inconvenient. Commonly instead one acquires time-to-failure information and proceeds to an initial specification of the time-to-failure probability density function f(t). This is typically a peaked function that increases as hardware vulnerability increases and decreases as significant numbers of the test population are lost to failure. When f(t) has been characterized the corresponding reliability and hazard functions can be obtained analytically or numerically from Eqs. (1a) and (3b), respectively. A commonly encountered wearout time-to-failure distribution is the Gaussian or normal distribution

$$f_N(t) = \left(1/\sigma_t \sqrt{2\pi}\right) \exp\left[-\frac{1}{2}\left(\frac{t-\mu_t}{\sigma_t}\right)^2\right] \tag{11}$$

having mean value or position $\mu_t$ and spread or dispersion $\sigma_t$. In corrosion problems the times-to-failure of similar units are often log normally distributed (i.e. the logarithms of the times-to-failure are distributed normally). The log normal distribution is

$$f_{LN}(t) = \left(1/\sigma_{lnt} t\sqrt{2\pi}\right) \exp\left[-\frac{1}{2}\left(\frac{\ln t - \mu_{lnt}}{\sigma_{lnt}}\right)^2\right]. \tag{12}$$

Examples of Eqs. (11) and (12) and their corresponding reliability and hazard functions are plotted in Figures 3 and 4.

## 2.4    Infant Mortality

Infant mortality refers to the early failure of substandard hardware items. These units contain flaws or defects not properly representative of the entire population of nominally similar devices. As early failures occur weaker units are removed from service while more rugged ones continue to function. Thus, as the early phase progresses the probability per unit time of experiencing additional failures decreases. Early life is characterized by a decreasing hazard rate, a decreasing time-to-failure density function, and a reliability function that decreases more rapidly than an exponential function at first and then more slowly.

4

Examples of the infant mortality hazard, reliability, and time-to-failure
probability density functions are presented in Figure 5. Quite often early
failure studies will be best represented by time axis displacements correspond-
ing to failures having occurred during manufacture or transit or otherwise prior
to the initiation of actual reliability testing.

Figure 5 refers to a purely early failure situation. That is, all units
are taken to be substandard for illustrative purposes. Normally a test popula-
tion will contain both normal devices and congenitally weak units. The latter
are potential early failure candidates and may be largely separated and prevented
from causing subsequent service problems by appropriate preliminary exercising or
burn-in procedures. After burn in the remaining population of hardware items can
be characterized as exhibiting purely random hazard, purely wearout, or perhaps
combined random and wearout behavior. The minimum vulnerability to normal load
stresses (force, pressure, voltage, current, temperature, etc.) occurs during the
random hazard or exponential reliability phase. Thus, for critical applications
early failures must be systematically eliminated (via burn in or proof testing,
for example). Similarly, the impact of wearout must be ameliorated by proper
parts selection, adequate design measures, and through preventive maintenance.

Early failures are most conveniently represented using the Weibull distribu-
tion, a subject that is deferred to Section 3.1.

## 3.0 FURTHER DISTRIBUTIONAL ASPECTS

In the previous sections of this report we began to touch upon the statistical aspects of reliability and service life of hardware. Times-to-failure were found to be distributed. A few important distributions are in common use to describe several important behavior classes (random, early, and wearout failures). There are many other well established distributions that are useful from time to time in reliability work. However, for the present purpose it is necessary to limit the scope of discussion here.

Section 3.1 deals with Weibull statistics, a generalization capable of unifying the descriptions of the random hazard, wearout, and infant mortality situations. In Section 3.2 we introduce the idea that reliability itself must be distributed. The connection of distributed time-to-failure properties with underlying system complexity and functional redundancy is touched on in Section 3.3 and its subsections.

## 3.1 A Generalized Description--Weibull Statistics

In 1951 Weibull[1] introduced a probability density function which has proved to be very comprehensive and well suited to reliability and life studies. There are both 2 and 3 parameter versions of the Weibull distribution. The three-parameter probability density function is

$$f(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1}\exp\left(-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right) . \tag{13}$$

The corresponding Weibull reliability and hazard functions are

$$R(t) = \exp\left(-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right) \tag{14}$$

and

$$\lambda(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1} . \tag{15}$$

The allowed ranges of the parameters are:

$$\gamma \leq t \leq \infty$$
$$-\infty < \gamma < \infty$$
$$\eta > 0$$
$$\beta > 0 .$$

6

The two-parameter Weibull reliability model is obtained from Eqs. (13) through (15) by setting the location parameter $\gamma$ equal to zero. Equation (15) represents equally well decreasing, constant, or increasing hazard rate situations as the parameter $\beta$ takes values less than, equal to, or greater than one respectively. Since $\beta$ has such a dramatic impact on the character or functional shape of the Weibull distribution, it is referred to as the shape parameter. Weibull shape effects are illustrated in Figure 6. Changing only $\eta$ has the same visual effect on a plot of the distribution as stretching or compressing the abscissa coordinate scale and symmetrically compressing or stretching the ordinate scale. This leaves the normalization of Eq. (13) unaffected. Thus, $\eta$ is referred to as the scale parameter of the Weibull distribution.

We have seen that the Weibull distribution has a decreasing, constant, or increasing associated hazard function depending on parameter choices. Another way to verify the versatility of this model is to look at limiting forms of the Weibull distribution function itself. Reference 2 and sources referred to therein point out that for $\beta = 1$, 2, and 3.313 Eq. (13) reduces respectively to the two-parameter exponential distribution, the Rayleigh distribution, and approximately to the normal distribution. The common one-parameter exponential distribution obtains when $\beta = 1$ and $\gamma = 0$. Equation (13) is skewed to the right for values of $\beta$ up to about 3.313 and skewed to the left for $\beta$ greater than 3.313. In the former case ($\beta < 3.313$) it is likely that a choice can be made such that Eq. (13) is also a satisfactory representation of the log normal distribution.

The Weibull distribution is very convenient in that it allows the same formal reliability description to embrace all three important practical situations. However, infant mortality, random failures, and wearout are not represented simultaneously by the same Weibull distribution. When more than one type of failure mode operates in a group of items of interest, the group is referred to as a mixed population. The reliability description of mixed populations is discussed further in Section 5.2.3. Estimating the parameters of the Weibull or other distributions is dealt with specifically in Section 6.2.

## 3.2   Distributed Reliability--Confidence Limits

Reliability has been introduced as a quantity measuring the probability of successful operation of a given component or system under specified conditions. Reliability is completely specified as a deterministic function of time (and loading and strength parameters) via Eqs. (1a) and (1b) if an appropriate time-to-failure probability density function $f(t)$ or hazard function $\lambda(t)$ is supplied. We have no trouble reconciling the concept of deterministic reliability with the variability of success/failure outcomes when we actually operate equipment. Reliability is only the probability of success and not a guarantee of successful operation in some fraction of attempts made.

The situation can be likened to the casting of dice. If a die is formed symmetrically, we assign it an a priori probability of 1/6 of showing any one of the six faces when cast. Even if the die is fair (unloaded), however, this does not assure that in six throws each face will show a single time only. But in a large number of throws the fractional exposure of each face of the die will

7

approach 1/6 for a fair die. If the die is loaded, different occurrence fractions for the six faces will be obtained in this way. We will have measured the loading in terms of the unsymmetrical a posteriori probabilities of showing the six die faces. Whether the die is fair or not, the number of times a given face shows up in a certain number of throws is a random variable subject to fluctuations under a repetition of the test. Thus, the best one can hope to do is to characterize the situation in terms of observed averages and some measure of the scatter of the data used to form them. This is a distributional description and the probability of showing any particular die face is distributed. In the case of a fair die the distributed a posteriori probability will have a high probability of including the a priori value. If the die is loaded, the former will likely exclude the latter.

How is reliability to be compared and contrasted with the casting of dice? First of all, a posteriori probabilities are measured in the same way--by operating the equipment and counting successes and failures or by casting dice and similarly noting the outcome. Many dice can be used or a single die can be thrown repetitively. Similarly, many (identical) equipments can be exercised or a single one subject to appropriate repair between uses. An important contrast is that in general there is no suitable way to assign an a priori reliability. Previous experience with similar equipment constitutes a related measurement rather than an evaluation based on structural arguments and advanced independently of operational experience.

At the beginning of this section we noted that reliability is fully determined if the related time-to-failure or hazard functions are completely specified. One can invent reliability models where this is imagined to be the case, but as a practical matter this situation does not occur. In practice the properties of the continuous functions $f(t)$ or $\lambda(t)$ are inferred from a limited number of discrete observations. The result is that only a statistically distributed description of the parameters of these functions can be specified. The derived reliability function is also distributed. Similar reliability conclusions are drawn directly if one focuses attention on the unfailed fraction of an equipment population as a function of time rather than the equivalent indicators (observed failure times, number of failures in intervals of equal duration).

In order to further clarify the concept of distributed reliability, let us pursue a more formal line of reasoning. To be specific and restrict the scope of the discussion somewhat, consider a nonreplacement test of N equipments which is terminated at the occurrence of the $r^{th}$ observed failure. Each of the r failure times $t_1,\ldots,t_r$ is recorded. Let us consider further for the moment that we have some independent assurance that the equipments under test are identical and exhibit exponential reliability. (In nature radioactive or fluorescent atoms of the same kind meet these last two requirements--for manufactured hardware, however, this must be recognized as an idealization.) We take our problem to be specifying the parameter $\theta$ of the one-parameter exponential time-to-failure distribution

$$f(t) = \frac{1}{\theta} e^{-t/\theta} \qquad (16)$$

8

from the available set of observed $t_i$ ($i = 1,\ldots,r$). Notice Eq. (16) is simply Eq. (9) written in terms of $\theta = 1/\lambda = $ MTBF.

On the basis of maximum likelihood theory the best estimate $\hat{\theta}$ of the true MTBF $\theta$ is (see Appendix 10.C.1 of Ref. 3, for example)

$$\hat{\theta} = \frac{1}{r}\left[\sum_{i=1}^{r} t_i + (N-r)t_r\right] . \tag{17}$$

The estimator $\hat{\theta}$ is in fact a distributed random variable since application of Eq. (17) to more than one nominally identical experiments will yield different results. Many such repetitions would produce an experimental determination of the distribution of estimators $f(\hat{\theta})$. Since we are dealing with a known time-to-failure distribution [Eq. (16)], this information can also be developed analytically. In the pioneering study in this area Epstein and Sobel[4] have shown that the distribution of estimators based on observing r failures among N units drawn from an exponential population is

$$f(\hat{\theta}) = \left(1/(r-1)!\right)\left(r/\theta\right)^r(\hat{\theta})^{r-1}\exp(-r\hat{\theta}/\theta). \tag{18}$$

The average or expected value of $\hat{\theta}$ is

$$E(\hat{\theta}) = \int_0^\infty \hat{\theta} f(\hat{\theta})d\hat{\theta} = \mu_{\hat{\theta}} = \theta . \tag{19}$$

Similarly

$$E(\hat{\theta}^2) = \left(\frac{r+1}{r}\right)\theta^2 . \tag{20}$$

The variance is

$$\sigma_{\hat{\theta}}^2 = E(\hat{\theta}^2) - \left(E(\hat{\theta})\right)^2 = \theta^2/r . \tag{21}$$

And the coefficient of variation is

$$COV = \sigma_{\hat{\theta}}/\mu_{\hat{\theta}} = 1/\sqrt{r} . \tag{22}$$

As an example Eq. (18) is plotted in Fig. 7 for the case $r = 10$, $\theta = 1$. The cumulative of Eq. (18) is also shown in Fig. 7 from which we see for example that the 90% two-sided confidence statement that can be made with respect to the expected range of $\hat{\theta}$ is

$$0.52\theta \leq \hat{\theta} \leq 1.56\theta . \tag{23}$$

9

Equation (23) states that if exponential units having true MTBF $\theta$ are tested until 10 failures are observed, the estimator $\hat{\theta}$ will have a 90% probability of being in the indicated range. Normally, however, the true $\theta$ is unknown and we would like to reverse the philosophy of Eq. (23) to sharpen the description of $\hat{\theta}$ as an estimator of $\theta$.

We turn again to the work of Epstein and Sobel[4] who showed that the random variable

$$z = \frac{2r\hat{\theta}}{\theta} \tag{24}$$

is $\chi^2$ distributed with 2r degrees of freedom. That is

$$f(z) = \left(1/2^r (r-1)!\right)\left(z^{r-1}\right)e^{-z/2} . \tag{25}$$

Using the variable transformation methods described in Chapter 5 of Ref. 5, Eqs. (18) and (25) are seen to be trivially related. Via reasoning along the same line (the process is amplified in Appendix B), one obtains the distribution of true MTBF values $\theta$ compatible with a single observed estimate $\hat{\theta}$. Thus

$$f(\theta) = \left(1/r! \hat{\theta}\right)\left(r\hat{\theta}/\theta\right)^{r+1} \exp(-r\hat{\theta}/\theta). \tag{26}$$

The origin moments of Eq. (26) of interest are

$$E(\theta) = \left(\frac{r}{r-1}\right)\hat{\theta} = \mu_\theta \tag{27}$$

and

$$E(\theta^2) = \frac{r^2 \hat{\theta}^2}{(r-1)(r-2)} . \tag{28}$$

The variance and coefficient of variation are

$$\sigma_\theta^2 = \frac{r^2 \hat{\theta}^2}{(r-1)^2(r-2)} \tag{29}$$

and

$$COV = \sigma_\theta / \mu_\theta = 1/\sqrt{r-2} . \tag{30}$$

Equation (26) together with its cumulative is plotted for the particular case $r = 10$, $\hat{\theta} = 0.9$ in Fig. 8. The cumulative of Eq. (26) can be used directly to make any desired confidence statement with respect to the ranging of $\theta$ about $\hat{\theta}$ for a given r. For example, from Fig. 8 the 90% two-sided confidence statement is

10

$$0.56 \leq \theta \leq 1.65. \tag{31}$$

Or since $\hat{\theta} = 0.9$ an equivalent representation is

$$0.62 \, \hat{\theta} \leq \theta \leq 1.83 \, \hat{\theta} \, . \tag{32}$$

The approach described in the preceeding paragraph is awkward since Eq. (26) must be integrated for each pair of experimental parameters $\hat{\theta}$, and r. Thus, standard practice involves instead the combined use of Eqs. (24) and (25) and available tables based on the $\chi^2$ distribution. The probability statement on z at a confidence level of $1-\alpha$ is

$$P\left[ \chi^2_{(1-\alpha/2),2r} \leq \frac{2r\hat{\theta}}{\theta} \leq \chi^2_{\alpha/2,2r} \right] = 1 - \alpha \, . \tag{33}$$

An equivalent statement providing the two-sided confidence limits on $\theta$ at the $1-\alpha$ confidence level is

$$L_2 = \frac{2r\hat{\theta}}{\chi^2_{\alpha/2,2r}} \leq \theta \leq \frac{2r\hat{\theta}}{\chi^2_{(1-\alpha/2),2r}} = U_2. \tag{34}$$

One-sided confidence limits are implied by Eq. (34) under the replacement $\frac{\alpha}{2} \to \alpha$ (for L or U but not both since the conjugate limit moves off to $\pm\infty$).

Since there is a one-to-one correspondence between MTBF and exponential reliability via Eq. (8), and using Eq. (34) the confidence interval for the reliability function may be specified as

$$e^{-t/L_2} \leq R(t) \leq e^{-t/U_2} \, . \tag{35}$$

Of course the full distributional character of the reliability function can be displayed by applying the methods of Appendix B to Eq. (26) with the proviso

$$R(t) = e^{-t/\theta} \, . \tag{36}$$

This yields

$$f(R) = \left( 1/(r-1)! \right) \left( r\hat{\theta}/t \right)^r \left( -\ln R \right)^{r-1} R^{(r\hat{\theta}/t - 1)} \, . \tag{37}$$

Equation (37) is plotted in Fig. 9 for the cases $r = 10$, $\hat{\theta} = 0.9$ and $t = 0.1$, 0.2. One notices that the reliability dispersion as well as its mean value is time dependent. Another way of visualizing this situation is displayed in Fig. 10 which shows the time dependence of the 50% confidence boundary and 80% two-sided confidence limits on reliability for the example being considered.

In closing this section let us consider a practical procurement example. The Navy recognizes that perfection of hardware performance and its specification

11

are both unattainable on a finite budget. Thus, compromises in both areas are commonplace in deference to the recognized statistical character of reliability. One might require that the reliability of some equipment for a specified time period be at least 90%. One might further demand sufficient testing under actual service conditions to support this statement at a 90% level of confidence. The problem is a standard one of specifying a one-sided confidence level and limit. The desired lower, one-sided confidence limit on the reliability is 0.9 and the confidence level is 90%. Combining Eqs. (34) and (35) specialized to this case yields

$$R(t) \geq \exp\left(-(t/2r\hat{\theta})\chi^2_{\alpha,2r}\right). \tag{38}$$

Equation (38) can be rearranged to give the minimum MTBF estimator $\hat{\theta}_{min}$ needed to satisfy the reliability specification at a confidence level of $1-\alpha$. Thus

$$\hat{\theta}_{min} = \frac{t\chi^2_{\alpha,2r}}{2r\left(-\ln R(t)\right)} . \tag{39}$$

Equation (39) expresses the minimum observed MTBF [via testing per Eq. (17)] needed to assure with $100(1-\alpha)\%$ confidence that reliability of at least $R(t)$ is achieved by an exponentially reliable system for a mission of duration t. This result depends on the number of failures r on which the MTBF estimate is based and hence on the level of testing to which one is willing to commit. Let us return to the specific example $(1-\alpha) = 0.9$, $R(t) = 0.9$ and use Eq. (39) to plot $(1/t)\hat{\theta}_{min}$ as a function of observed failures r. This result is shown in Fig. 11. Figure 11 includes similar results for a few other confidence levels also. One can see generally that if $\hat{\theta}$ is supported by 5 to 10 observed failures the lower confidence limit $\hat{\theta}_{min}$ must be roughly 15 times the desired mission duration for 90% reliability at a 90 to 95% confidence level.

## 3.3    A Components Versus System View

As consumers and users of products we usually take a systems view of reliability. We ask "Is the car running?" rather than inquiring separately about the operational health of the tires, battery, fuel pump, engine seals, hydraulic and electrical subsystems, etc. But as engineers, scientists, and managers charged with improving sonar transducers we need to focus attention on specific areas where constructive changes would have a beneficial impact. We tend to think of systems as assemblages of components. As we shall see shortly, this posture probably relates more to trends in commercial packaging than distinctions relating to form and function. For example, a simple square wave oscillator might be a system of interest. It is assembled from components such as an integrated circuit, resistors, capacitor, battery, switch, printed circuit board, etc. The capacitor is a component because it is purchased as a separate commodity (No one buys tin foil and paper and rolls his own capacitors anymore.). But to the manufacturer of capacitors this device is itself a system assembled from a variety of more homogeneous materials. Even in looking at this indenture level we have overlooked the processing steps required to convert naturally occurring raw materials into the conductive foils and wires and insulating films that go into capacitor construction.

12

Similarly, the integrated circuit referred to is purchased, installed, consumed, and replaced as a separate item (component). But with respect to internal form and function this device is a system of high architectural complexity. Many very carefully controlled masking and metallurgical processing steps are involved in its manufacture. The system is complicated (and inexpensive) enough to defy practical diagnostics and repair. Systems which are more expensive to repair than replace tend to receive modular packaging and be treated as throw-away items. Curiously then it is economics and not structural complexity that most strongly influences the component versus system distinctions that we normally draw. An automobile oil filter is a throw-away item not because it is structurally simple or complex but because it is easier and cheaper to replace it than to clean and evaluate it for reuse.

We have just seen that with respect to form and function commercial components may in fact be exotic microsystems. Conversely many heroic structures have a very simple functional makeup. Structurally, a highway is scarcely more than a ribbon of aggregate material. The members of a bridge or a barge are more homogeneous or less functionally diversified than a simple capacitor or battery. Furthermore, the former are considered repairable while the latter are not.

Empirical reliability studies tend to attempt to catalog components-level experience from which systems-level descriptions are built by superposition. Looking at dictionary definitions, one finds "component" referred to as a constituent part while "system" means an assemblage of such parts. One perceives that components are to be thought of as structurally simple while systems are complex. Often, however, the reverse is true as we have seen. We shall explore further the reliability implications of this structure dichotomy in the next two sections of the report.

## 3.3.1    Complexity and Redundancy

In the previous section we have begun to see that the simplicity or complexity of a fabricated item is not necessarily related to whether we regard the item to be a component, system, subsystem, etc. Why do we wish to make such distinctions? This is because reliability is related to features of true form and function rather than to arbitrary packaging and assembly constraints. The two important reliability classes of interest here—exponential and wearout—are primarily associated with functional nonredundancy and redundancy respectively. Complex systems usually (but not always) exhibit little redundancy and are exponentially reliable. Simple structures tend to have an excess of working material and so are functionally redundant in a way that leads to wearout reliability or a strongly peaked time-to-failure distribution.

Consider an ordinary steel tensile member. This may be a modern marvel metallurgically. But from a reliability standpoint it is homogeneous with many interatomic bonds sharing the applied load. If the tensile member is conserva- tively designed, many bond failures or considerable loss of material (to abrasion, oxidation, corrosion, etc.) can occur before catastrophic failure results. This situation is in contrast to that exhibited by a complex system the operation of which depends on the simultaneous functioning of many subsystem-structures. In the latter case, many failure modes whether they individually exhibit exponential

13

reliability or not contribute randomly to system unreliability. As a result systems tend to be characterized by exponential or random hazard reliability. In contrast redundant structures tend to exhibit wearout reliability. We can consider then the appropriateness of exponential reliability modeling for electronic components. If such components were simple redundant structures, they would be expected to exhibit wearout reliability. If they are in fact complicated microsystems, then one would expect their reliability description properly to be exponential. The latter situation seems to be the one observed and is certainly the basis of contemporary handbook reliability prediction. There is also the implication that components that exhibit true internal redundancy be separated by class and modeled appropriately (wearout reliability).

It has long been recognized that mechanical devices do not fit the exponential modeling pattern as neatly as do electronic components. Hopefully, the reasoning of the last few paragraphs provides some relevant insights. How then should sonar transducers be properly treated? Structurally they are relatively uncomplicated. One would look more for wearout failure modes than random hazard vulnerabilities. This approach is in contrast to most transducer reliability modeling efforts, which thus far have attempted to apply the purely exponential approach borrowed from electronics reliability.

The relationship of reliability to system or component complexity, simplicity, or redundancy features is subject to various confounding influences in practical situations. Caution is advised as suggested by the examples presented in Section 3.3.2.

3.3.2    Further Confusion--Examples

We have already noted some of the redundancy features of material used in bulk. These properties depend largely on configuration, however. Consider doubling the amount of material in a tensile member, for example. If this involves preserving the length while doubling the section area, the loading performance and vulnerability to damage are both significantly improved. If the length is doubled and the section area preserved, one expects a slight worsening of tensile strength because the probability of encountering a performance limiting flaw is doubled.

A capacitor also seems to be a simple system involving a dielectric film placed between conductive foils. This is not really a bulk application, however, since any single flaw in the dielectric can lead to voltage breakdown of the device. Again, configuration details are important. If the area of the dielectric is doubled while its thickness is kept the same, performance (capacitance) and vulnerability both increase. If the dielectric area is fixed while the thickness is doubled, capacitance and susceptibility to voltage breakdown are both reduced (assuming that the operating conditions are not changed). The reliability benefits are due to reduced specific loading rather than an ability to tolerate material damage. The capacitor remains a nonredundant, exponentially reliable device. In contrast, the tensile member can survive material damage, exhibits redundancy and wearout reliability.

Redundancy can be artificially introduced into a reliability problem by providing backup systems in one form or another. When this is done, the overall system will exhibit classical wearout reliability even if all the subsystems

14

involved exhibit purely exponential reliability.  Consider an example.  Suppose a sonar array consists of 100 transducer elements each taken to exhibit exponential reliability $R_e$ where

$$R_e = e^{-\lambda t} . \tag{40}$$

Further, imagine that acceptable beam forming and acoustic signal recovery characteristics are achieved if 90 or more of the 100 elements are functional (This is our system success/failure criterion.). The probability of finding exactly m failed units among N total identical devices of reliability $R_e$ is binomially distributed via

$$P_m = \frac{N!}{m!\,(N-m)!} \, R_e^{(N-m)} \left(1 - R_e\right)^m . \tag{41}$$

More explicitly using Eq. (40)

$$P_{N,m}(\lambda t) = \frac{N!}{m!\,(N-m)!} \left(e^{-\lambda t}\right)^{N-m} \left(1 - e^{-\lambda t}\right)^m . \tag{42}$$

Equation (42) is a discrete probability density function which for all t meets the test $\sum_{m=0}^{N} P_{N,m}(\lambda t) = 1$.  Acceptable system performance is associated with the occurance of 10 or fewer element failures.  Thus, we can define system reliability as

$$R_s = \sum_{g=0}^{m} \frac{N!}{g!\,(N-g)!} \, R_e^{(N-g)} \left(1 - R_e\right)^g = PBS(N,m,t) . \tag{43}$$

Equation (43) has the form of a partial binomial sum [hence the notation PBS(N,m,t)].  The system reliability given by Eq. (43) for the case N = 100, m = 10 is plotted in Fig. 12a.  The time-to-failure probability density function associated with Eq. (43) is given by $f_s(t) = -dR_s/dt$ [Eq. (3c)].  Again, using Eq. (40) and performing the indicated differentiation we find

$$f_s(t) = (N-m)\lambda P_{N,m}(\lambda t) . \tag{44}$$

This function is a continuous probability density function satisfying $\int_o^\infty f_s(t)dt = 1$.

Equation (44) is plotted in Fig. 12b for the case of interest (N = 100, m = 10). Comparing Figs. 12 and 3 or 4 we see that the transducer array exhibits reliability features characteristic of wearout.  This is purely the result of allowing component redundancy in the system success criterion.  The transducer elements themselves were taken to have exponential reliability.

Let us consider an example of just the opposite situation--a case where components subject to wearout alone lead to exponential system reliability. Bazovsky[6] has treated just such a problem in examining the impact of making replacements only as failures occur within a population of 10,000 light bulbs.

15

In the interest of overcoming a slight oversimplification in Bazovsky's[6] treatment, let us reanalyze a similar example. Consider a group of N incandescent lamps subject to wearout failures only. Take the time-to-failure distribution to be normal centered at $\mu$ with standard deviation $\sigma$ as shown in Fig. 13a. We imagine that as each lamp fails it is replaced by a new one. Thus, when all the original population has dropped out of service it has been replaced by a second generation of lamps. However, these units have been placed in service over a range of times rather than all at once. As a result, the time-to-failure distribution will now show greater dispersion than that due to wearout effects alone. This convolution problem is analyzed in Appendix C. The dispersion effects continue to grow with each replacement generation. Soon different lamp generations are represented at the same time. This situation is shown in Fig. 13b. The superposition of the individual time-to-failure distributions representing various lamp generations is depicted in Fig. 13c. This function oscillates at first and then gradually settles to a constant value of $N/\mu$. This equilibrium failure rate is usually unacceptably high. Thus, one cannot ordinarily tolerate replacing wearout failures as they occur. Rather, it is much more productive to anticipate wearout (via pilot studies) and engage in preventive maintenance a few time-to-failure standard deviations before the mean wearout life $\mu$.

In the example just discussed a constant failure rate develops because the replacement strategy invoked leads to units being placed in service at random times. This has the effect of completely masking the intrinsic wearout characteristic assumed to be operative. Another implication of the constant failure rate that develops is that the population will decay exponentially if the replacement program is abandoned. This is an example of exponential reliability associated with purely wearout failures. Contriving to display exponential properties within a pure wearout situation is not merely a pedantic exercise. Multiple test stand replacement testing is often carried out in evaluating the performance of exponentially reliable units. One is cautioned to observe the distribution of individual times to failure as well as the total number of failures and total test time. This permits the confirmation of a true random hazard situation and avoids confusion with the case where wearout units are placed in service at random times. Similar concerns arise in servicing commercial equipment. Experiencing a constant replacement rate for a particular component does not alone determine whether the failures involved are of the random or wearout type.

## 4.0    RELIABILITY PREDICTION

Reliability prediction is an exercise that one engages in prior to committing to the production of new hardware. The purpose of it is to estimate the probable survival characteristics of the equipment against mission objectives. A relevant dictionary definition states that to predict is to foretell with precision of calculation, knowledge, or shrewd inference from facts or experience. Thus a solar or lunar eclipse may be predicted by measuring the relative positions of the sun, earth, and moon; discovering the laws that describe their motion; and calculating trajectories for future time. Within the disciplines of physics and astronomy all of this has been elegantly accomplished. On the celestial scale position and time can be simultaneously measured to high precision. The problem is also well characterized by considering only gravitational forces. Astronomical prediction is considered to be mature, satisfying, and successful.

In a sense the philosophy of reliability prediction is the same as that of any predictive science--inferring some future behavior from past observations. Reliability prediction is also different in some ways than areas such as astronomical prediction. In reliability work one is not generally concerned with dynamics--evolution from an observable initial state via discoverable laws of behavior. (An exception to this statement is provided by the related area of failure analysis.) Reliability prediction usually attempts to draw inferences from similarities of a system of interest to hardware previously evaluated. The complex conditions of environment and use make more detailed treatments of reliability prediction problems very difficult. One can even argue that some kinds of reliability problems do not exhibit a failure dynamics of much interest. For example, in the random hazard situation one is interested in postponing catastrophic loss of function rather than examining its (rapid) development in time when it does occur. In contrast, of course, the dynamics of wearout phenomena are a major issue.

From earlier sections of the report we have seen that even under the most ideal conditions reliability information may be expected to exhibit a highly distributed character. Of course astronomical observations are also distributed although dispersion effects are often much less significant giving the latter an appealing flavor of determinism. This difference is not due to the inability of reliability studies to attract intellectual giants to play the roles Brahe, Galileo, Kepler, and Newton did for astronomy. Neither is it the result of a lack of substantial and sustained funding. Reliability prediction is an awkward and difficult discipline because of the diversity of objects of interest, the variability of environmental and use conditions, and difficulties in defining process endpoints. Nevertheless when the economic benefits of improved product performance are considered, reliability studies are found to be very cost effective.

In the following sections we consider briefly several relevant aspects of reliability prediction.

### 4.1    Original Impetus

Modern reliability studies as a formal discipline are generally taken to have originated with the German V rocket programs of World War II. Early

17

thoughts were that quality of manufacture would be a secondary issue in a device intended to see service of only some tens of minutes. The incorrectness of this line of reasoning was emphasized by the failure of virtually all of the rockets initially built. It was then recognized that to have even a moderate chance of performing satisfactorily, a complicated system must be built of very highly reliable components. The reliability of a system without redundancy is the product of the component reliabilities. Learning this lesson turned out to be one of the prerequisites to entering the space age.

When reliability concepts began to be introduced in connection with military procurements in this country, one of the first aggravating dispersion effects in this field surfaced. Different contractors bidding on the same job would predict substantially different reliabilities for their versions of the desired product. Complicating the situation was that these conclusions were developed by using a variety of different unstandardized sources of supporting information. The climate was one that did not permit easy evaluation of the relative merits of competing proposals for the same work. Thus a major interest of the government in supporting the development of universal reliability prediction methods was to put competing contractors on equal terms. If all bidders were using the same comprehensive source of reliability data in the same way, it was reasoned that superior predicted reliability would be a reflection of a better hardware design. Ambitious as it sounds such a scheme has been implemented. We now have a variety of military standards, handbooks, and procedures in place providing instruction for the uniform disposition of reliability questions relating to procurements. Putting competing contractors on an equal footing has been pretty well accomplished. In fact the heroic and well maintained edifice of reliability prediction tools has become so thoroughly entrenched that its users have largely forgotten its origins. There is a tendency among procurement managers to view reliability prediction as a mature and promising approach to obtain the kinds of answers they need for solving hardware supply problems. Reliability studies are beneficial but they often fall short of the expectations people outside the field have for them.

We have already seen that reliability prediction has succeeded in a relative way by equalizing the procurement process. Not much work has been done in evaluating the absolute success (How well does prediction compare with measurements on the same equipment?) of reliability prediction. In one case that has come to this author's attention involving avionics radio equipment, prediction of the system MTBF produced values ranging from 2 to 10 times larger than those subsequently measured.[6] Clearly this level of correlation does not allow prediction to be substituted for actual in-service measurements if a meaningful hardware evaluation is desired.

## 4.2    Current Practice

As presently implemented, reliability prediction generally takes one of two basic forms. The standard handbook approach is very commonly used in the military hardware procurement setting for which it was developed. The other major reliability discipline that lends itself to performance prediction is called probabilistic design. From the user's viewpoint handbook reliability prediction represents a sort of cookbook approach to the problem. Probabilistic design is

18

more analytical, distributional in focus, and academic in flavor. One should not feel that one approach is correct and the other isn't. Handbook prediction at times seems to lack rigor but nonetheless can be applied to rather complicated problems. Probabilistic design can be quite definitive for small scale problems but often is prohibitively difficult or preempted by information gaps in larger settings. We will look further at some of the basic features of these two approaches in the next two subsections of the report.

### 4.2.1   Handbook Methods

Under the handbook methods heading we will limit discussion to topics dealt with in the dominant source work in this field--MIL-HDBK-217C.[7] The basic nature and use of handbook techniques and information will be considered. It is also well to be aware of the proper scope and limitations of handbook prediction. Reference 7 addresses the latter point in Section 1.3. Electronic components and systems are taken in this setting to be exponentially reliable. Thus failure rates are additive and time independent.

MIL-HDBK-217C is the latest revision of the most definitive document relating to the problem of correlating the observed and expected behavior of important classes of electronic systems and components. It summarizes in tabular form vast quantities of data accumulated under actual field service conditions. Most of MIL-HDBK-217C treats a reliability prediction method called "Part Stress Analysis". This is a rather detailed kind of description requiring complete design and operating information relating to the system of interest. Implementing this approach requires one to know a great deal about prevailing thermal conditions, electrical loading, and the service environment generally. Assuming these identifications can be made, the handbook provides associated failure rate information either in tabular or analytic form. Generally speaking, the desired reliability information is structured as a base failure rate modified by environment, quality, use, etc. factors. The base failure rate incorporates temperature and primary electrical effects and is specific to device category. The modifiers are multiplicative quantities called $\pi$-factors. In virtually all component categories the environmental and quality factors $\pi_E$ and $\pi_Q$ appear. A variety of other $\pi$-factors generally also occur. For example, the part failure rate model for general purpose diodes is expressed as

$$\lambda_p = \lambda_b \left( \pi_E \times \pi_Q \times \pi_R \times \pi_A \times \pi_{S2} \times \pi_C \right) \text{ failures}/10^6 \text{ hours.} \tag{45}$$

The subscripts R, A, S2, and C refer to current rating, application, voltage stress, and construction respectively. The base failure rate for discrete semiconductors (including diodes) is represented as

$$\lambda_b = A \left[ \exp\left( \frac{N_T}{273 + T + (\Delta T)S} \right) \right] \left[ \exp\left( \frac{273 + T + (\Delta T)S}{T_M} \right)^P \right], \tag{46}$$

where

19

A is a scaling factor.

$N_T$, $T_M$, and P are shaping parameters.

T, $\Delta T$, and S are temperature and thermal and electrical

      stress derating factors.

The base failure rate and the $\pi$-factors for each device category treated are
all presented in MIL-HDBK-217C. As an example of the range of variation possible,
the environmental and quality $\pi$-factors for general purpose diodes are presented
in Table II. The usual eleven environmental stress levels and five levels of
component quality are displayed. One can see from considering these two factors
alone that corrections to the base failure rate can be hundreds of times larger than
the base rate itself.

MIL-HDBK-217C displays a variety of analogs to Eq. (45) relating to electronic
components other than diodes. The versatility in this form of description comes
in the introduction of the wide range of $\pi$-factors relating to diverse properties
affecting the performance of different classes of components. Similarly Eq. (46)
is only a representative case. Other models are given in Ref. 7 relating to dif-
ferent situations. In reliability work descriptions such as Eq. (46) are called
"physics-of-failure" models. To a physicist this language is a little heady,
suggesting model development based on derivations from first principles. Actually
failure rate models should more properly be thought of as phenomenological
characterizations. Forms have been developed that with the adjustment of relatively
few parameters allow a large amount of field experience to be cataloged and
summarized. There is no need to apologize for this situation. Thermodynamics
is largely a phenomenological discipline. The latter also has a proper microscopic
basis in statistical mechanics, of course. One can think in terms of exploring
reliability problems more fundamentally with a view toward correlating cause and
effect. This is the failure analysis approach which occasionally is invaluable.
It must be used sparingly, however, in order to keep the scope of the overall
problem within tractable bounds.

The electronic components for which handbook reliability data sources have
been developed are treated by generic class. There are many hundreds of junction
transistor types that carry distinct part numbers. These are not distinguished
for handbook reliability purposes--they are all simply Group I discrete semicon-
ductors. Obviously then handbook reliability data is class average information.
In the handbook setting measures of dispersion within classes are never reported.
Similarly the user is never made aware of how much test/service time and how
many observed failures support reported failure rates. Thus one is not in a
position of being able to make statements about the statistical quality of hand-
book prediction. This is consistent with the view of the authors of MIL-HDBK-217C
who disclaim the absolute accuracy of handbook predictions while maintaining
their relative usefulness in the parallel procurement setting. Obviously the
handbook user is not being misinformed. One can argue, however, that he is left
seriously uninformed by a method that suppresses and fails to pass on available
dispersion information.

Thus far in this section we have discussed the Part Stress Analysis, or
more detailed type of handbook reliability description. Its implementation calls
for a mature system design and rather complete knowledge of component electrical

stresses, power dissipation, thermal and mechanical loads, etc. During the preparation and evaluation of contractor proposals and early product design phases, much of this information is not available. In this setting the "Parts Count" reliability prediction method is often employed. Here one need only identify components by generic type, quantities, quality levels, and the intended operating environment. The total equipment failure rate is given by evaluating the sum

$$\lambda_{EQUIP} = \sum_{i=1}^{n} N_i \, (\lambda_G \pi_Q)_i \; , \tag{47}$$

where

$\lambda_G$ = failure rate for the $i^{th}$ generic part

$\pi_Q$ = quality factor for the $i^{th}$ generic part

$N_i$ = quantity of $i^{th}$ generic part

$n$ = number of different part categories.

Equation (47) applies to a single operating environment. If different sections of the equipment operate in different environments, then partial sums of the form of Eq. (47) should be performed on a per-operating-environment basis and added. The parameters of Eq. (47) are tabulated in Section 3.0 of MIL-HDBK-217C. Testimony to the relative simplicity of this approach is that exposition of the method and presentation of all the supporting material requires only 10 pages of text.

If as we have seen the accuracy of Part Stress Analysis reliability prediction is suspect, then it is necessary to approach Parts Count results with still greater skepticism. This is true because a great deal of relevant information relating to use conditions is simply not available at this stage. Caution: If a contractor offers to just barely meet mission reliability objectives on the basis of a Parts Count prediction, let the buyer beware. Similarly, it is folly to contemplate substituting any form of prediction for a bona fide post-manufacture reliability verification study if one really wants to properly characterize hardware performance.

## 4.2.2    Probabilistic Design

Probabilistic design refers to a developing method of approaching reliability and related engineering design problems that emphasizes their statistical aspects. Every measurable engineering parameter is taken to be distributed rather than deterministic (having a single value only). The performance of an entity of interest is described in terms of the stresses it is subjected to and its strength or ability to function in a given stress environment. In quantifying this approach strength is defined simply as the stress level at which failure occurs. These terms are used here in a generalized sense. Thus stress may be electrical, thermal, mechanical, hygrometric, etc.--any relevant loading aspect of the situation of interest. Failure must also be adequately defined whether it be catastrophic, onset of irreversible damage, or some specified property degradation. Against this background reliability is defined as the probability that strength exceeds stress. For a system, of course, one has to ask this question simultaneously about every relevant stress/strength facet. The key to performing probabilistic design is to properly characterize the strength distributions of a piece of hardware and also identify from a distributional viewpoint the stresses operative.

21

Kececioglu[8] gives a detailed fifteen-step methodology charting how one might systematically grapple with a probabilistic design problem. Reference 3 also treats this topic in Chapter 4 with some minor variations from the approach presented in Ref. 8. The topical areas of Kececioglu's probabilistic design methodology are listed in Table III. It is beyond the proper scope of this report to attempt to convey to the reader a working appreciation of the probabilistic design method. However, an effort will be made to elucidate the underlying philosophy of the approach. Probabilistic design is basically a very detailed stress/strength overlap calculation. Reliability is simply the probability that strength exceeds stress under the conditions of the intended application. One can see from Table III that reliability prediction is one facet of probabilistic design. Normally, however, the focus is not on prediction of reliability but is directed toward tailoring design parameters to achieve a desired performance objective. In either case the price of measuring success can be quite high. Probabilistic design is a demanding discipline in terms of the quantity and quality of informational inputs required.

One should expect the probabilistic design approach to reliability questions to be ultimately compatible with relevant phenomenological descriptions. For example, exponential reliability is implied by a situation involving static and somewhat overlapping distributions of stress and strength. This leads to a constant vulnerability or failure probability per unit time or per load cycle. In contrast wearout is characterized by a monotonic loss of strength due to either fatigue under load or dissipative influences of the service environment. This increases the interference of stress and strength distributions leading to an increasing probability that additional service will result in failure. A specific example of corrosion wearout is treated probabilistically in Section 5.4. The early failure situation is also easily interpreted from the probabilistic design viewpoint. In this case the initial strength distribution is skewed to the left embracing substandard components. Application of normal service stresses leads to significant stress/strength overlap and a high probability of premature loss of function. In this case failures may also occur in transit or otherwise prior to being placed in service.

## 4.3    Limitations in the Sonar Context

We have just discussed the motivations for and some of the major developments in the area of reliability prediction. In recognition of the importance of product reliability and the successes of reliability studies in other areas, the Navy sonar community has taken steps to systematically improve sonar hardware through efforts having reliability as a specific focus. This kind of commitment has already produced beneficial results. Thus far, however, the benefits have been largely of a debugging nature--discovery of overt design or manufacturing defects--rather than the optimization of designs already established as workable. To those who felt that reliability prediction was already mature science (or art), recent progress in the sonar area has seemed painfully slow. There are several reasons that this should be so. Much sonar transducer reliability prediction work attempts a description from a components-level, random hazard point of view. This suffers from certain weaknesses. Sonar transducers are not assembled exclusively from components that are properly characterized by constant hazard functions. Wearout processes such as metal

fatigue and corrosion and water permeation of elastomeric materials are also operative. Many components are nonstandard from a reliability accounting point of view so that handbook failure rate source materials don't apply. This places the reliability data acquisition task in the hands of transducer production contractors or even the end user--the Navy.

A primary purpose of this report is to help the reader cultivate an appreciation of the enormity of the task of gathering meaningful reliability data for transducer systems. Why do transducers pose a particularly difficult problem? Several reasons. Transducers are intended to be long lived and opportunities for observation and maintenance of installed units are few and inconvenient. Transducers are a specialty item and production quantities are usually quite limited. These two factors combine to make it very difficult to gather together enough units to do statistically significant reliability testing. An even greater challenge is to produce reliability results in a timely fashion--when they can impact the hardware involved during design and development stages.

To this author's knowledge a complete, integrated transducer study advertised to be a probabilistic design evaluation has never been attempted. And yet many of the elements of a probabilistic design study are routinely developed by transducer acoustic design specialists and production engineers--persons whose focus is more on performance than reliability per se. Dynamic stress analyses of driven, mass-loaded piezoelectric ceramic and fatigue loading studies of stress rods and pressure release systems are examples. There are many case histories where this kind of evaluation has led to design changes or manufacturing adjustments associated with dramatic transducer reliability improvements. Significant progress is possible and has been achieved in areas where operational stresses and the strengths of component materials employed are both well characterized. The difficult situations are those where the properties of the materials involved change with time and temperature and perhaps loading history and the stresses operative have environmental origins and exhibit large fluctuations. Well developed mechanistic stress/strength overlap interpretations of phenomena such as water permeation, corrosion, and bond degradation have not yet been given. These are areas known to be important and hardware life-limiting in many situations. This provides a strong incentive but does not otherwise simplify the large task of assembling distributional information needed for probabilistic evaluation of these highly variable processes. Specific areas of difficulty are discussed in greater detail in subsequent sections of the report.

## 5.0    LIFE PREDICTION

In connection with characterizing the serviceability of hardware, the term
life refers to the entire period of useful, failure-free operation. Life as a
descriptive parameter is dual to reliability. Reliability is the probability of
realizing acceptable performance for a given period of time (some mission duration
or the period between overhauls for example). Life is the actual time that such
performance is achieved. Many similar equipments can be monitored to obtain a
distribution of lifetimes. Formally from Eqs. (2) and (6) of Table 1, reliability
is one minus the cumulative of the distribution of lifetimes. Conversely from
Eq. (3c) the distribution of equipment lifetimes is equal to the negative time
derivative of the reliability function. As with reliability notice the distribu-
tional flavor when we speak of hardware service life. We never inquire how long a
specific item will continue to operate. Rather we ask about its expected life or
the average or most probable lives of similar equipments. These are measures of
central tendency of some body of distributed information. If we are sophisticated,
we also look into the dispersion, asymmetry, etc. of the distribution when this
seems justified by the quality of sampling statistics.

Since service life and reliability are distributionally related, life
prediction and reliability prediction are really equivalent exercises. For
example, in doing handbook reliability prediction one arrives at a superposition
failure rate, inverts it to obtain the mean time between failures (MTBF), and
exponentiates the (negative) failure rate times time to obtain reliability. Thus
conjugate reliability and life information is developed simultaneously. If
reliability and life are so closely related, why do we address the two as separate
topics in this report? This is a good question. Within the context of reliability
theory the separation seems unnatural. But when practical concerns are raised
the reverse is true. Basically, whether equipment is used in a military,
commercial, or consumer setting, one is interested in two things--how well will
the hardware function and for how long? The answers to these questions enable us
to determine whether mission requirements will be met and what maintenance and
replacement costs and schedules will be. How well does equipment function (over
some specified time interval)? This is reliability. How long does it continue
to work? This is life.

The reliability/life dichotomy sorts itself out somewhat when we distinguish
random hazard and wearout effects. In the latter case times to failure often
tend to cluster so that any measure of central tendency is a reasonably descriptive
service life estimate. In contrast for the random hazard situation MTBF is a
poor measure of the broad exponential distribution of times to failure that one
expects to encounter. A much crisper description is obtained by specifying the
probability of surviving a mission of given duration, i.e., the reliability. One
can further interpret the random hazard situation as exhibiting a constant overlap
of the associated stress and strength distributions or an invariant vulnerability
to catastrophic damage due to extreme load fluctuations. The reliability parameter
is a measure of this. Wearout, on the other hand, is characterized by the
accumulation of damage until residual strength is commensurate with load stresses
encountered in normal service. Failure is inevitable and often with a very
predictable time scale.

24

Often a system of interest exhibits random hazard and wearout effects simultaneously. These may be independent or strongly coupled. For example, a capacitor may manifest an exponential voltage breakdown reliability characteristic and suffer fatigue wearout failures with thermal cycling. The former is a measure of the intrinsic resistance of the dielectric material to perforation under overvoltaging conditions. The latter may be due to improper lead dressing resulting in excessive flexure in service. These two effects are unrelated and we may speak of both a (voltage breakdown) reliability and a (mechanical fatigue) wearout life. The two effects can also be treated together in terms of either reliability or life concepts.

An automobile tire is an example of a system which exhibits coupled random hazard and systematic or wearout reliability aspects. The stress environment is due to ordinary road hazards—stones, chuckholes, railroad tracks, etc. However, the strength of the tire or its ability to survive exposure to the stress environment without sustaining damage is not static. Rather the strength decreases as tread material is worn away in normal service. Thus, the random hazard vulnerability increases as the wearout process proceeds. And of course if normal preventive maintenance steps were eschewed in this case, the tire would eventually succumb to a programmed wearout failure due to a random load stress.

The practice of speaking of reliability and life as if they were unrelated may stem from formulating separate treatments of the random hazard and wearout aspects of reliability problems. Life prediction must then emphasize wearout considerations. Let us bear this distinction as well as the formal unity of the subject matter in mind as we further explore the life prediction problem in the following sections of the report.


## 5.1    Definition of Life

We have already noted that life in reference to hardware is the period of useful, failure-free operation. For reliability evaluation purposes it is often necessary to be very specific about what constitutes acceptable performance. This may be easily accomplished such as in the case of an incandescent lamp for home lighting. If it lights when voltage is applied, it is good; otherwise it is considered failed. In the electric light case the transition between good and failed is usually quite abrupt corresponding to the evaporation of a portion of an old and weakened filament. Most hardware evaluation situations are more complicated and subtle. In a photo-processing application our electric lamp may have to be discarded when its intensity or spectral output fall outside acceptable limits rather than when the filament disintegrates. These two cases are examples of general classes of failure criteria: the sudden and complete loss of some physical function or the gradual migration of a performance property outside the normal useful range. In complicated equipment, of course, many subsystems must simultaneously meet appropriate performance tests. The more restrictive such situations are, the more difficult it is to be assured in practice that in-use equipment is performing adequately. For example, in sonar applications terminal resistance or drive power measurements do not provide detailed information on transducer efficiency or array beam-forming characteristics.

We have spoken of failure-free operation as defining the period of useful equipment life. This does not mean that no failures can be tolerated. Obviously

when a repair is effected, equipment is revitalized and can be returned to service.

One might be tempted in some circumstances to define service life to be that period of time when the hardware reliability remains above some specified level. This turns out to be circular since reliability is related to the time-to-failure probability density function or distribution of service lifetimes. Thus, it is necessary for individuals who draw up equipment specifications to decide very specifically what performance requirements need to be imposed. Reliability and service life are not defined except with respect to specified levels of performance (or complete definition of failure or non performance) under conditions of environment and use that are also fully characterized.

## 5.2    Some Dynamics of the Terminal Process

It is often convenient to identify the condition that corresponds to hardware failure (or the end of useful life) as irreversible damage due to some form of overstress. The transition from an unfailed to a failed state we choose to call the terminal process. This may occur rapidly with the application of an environmental overstress to a "good as new" structure. The vulnerability to rapid catastrophic failure may also build gradually via a wearout process associated with or incidental to normal use. Corrosion and fatigue have already been compared and contrasted from this point of view. In addition catastrophic change of state or loss of function need not occur at all for an item to be declared worn out. Automobile tires, for example, are ordinarily replaced in response to cues less dramatic than a flat or a blowout.

It is not our purpose here to engage in serious failure analysis. We shall avoid attempting to review specifically what can go wrong with the devices we build. It is of the utmost importance, however, to recognize that equipment can and will malfunction. This realization together with the motivation it stimulates to pursue sound design principles and constructive maintenance practices may be our best defense against unreliable hardware. Regarding the relative importance of these two features (design and maintenance) we can look to the complicated organic systems (including man) found in nature. Here repair is as dynamic and highly organized as creation itself.

## 5.2.1    Random Hazard Case

We have looked at the random hazard situation from a reliability point of view. Now let us consider this case from a perspective emphasizing stress/strength overlap and the mechanism of failure. The stress/strength overlap description of a random hazard problem is static in the sense that the strength distribution is taken to be fixed. This means, of course, that the conditions of use do not physically degrade the item of interest (until the ultimate catastrophic failure occurs). Also we are concerning ourselves with operation under uniform environmental conditions. A uniform environment is not one that does not exhibit variations. Rather it is statistically repetitive over time intervals of reasonable length. Shooman in Chapter 8 of Ref. 9 considers the application of stresses distributed randomly in time according to a Poisson distribution

$$g(n,t) = \frac{(\nu t)^n e^{-\nu t}}{n!} \tag{48}$$

to a part having a static strength represented by the distribution $f(S)$. Equation (48) gives the probability that n stresses will occur in a time interval of duration t. The amplitudes of these stresses are understood to be distributed as some $f(s)$. Shooman[9] calculates the probability that n stresses will occur and that the component of interest will survive all of them. Summing over all n yields the overall time dependent probability of success or component reliability

$$R(t) = e^{-Q\nu t}, \tag{49}$$

where Q is the static unreliability associated with a single probabilistic stress/ strength overlap encounter. Kececioglu and Cormier[10] have provided the analytical machinery for calculating Q and its complement the static, single stress cycle reliability $R = 1-Q$ via the expressions

$$R = \int_{-\infty}^{\infty} f(s) \left[ \int_{S}^{\infty} f(S) dS \right] ds = \int_{-\infty}^{\infty} f(S) \left[ \int_{-\infty}^{S} f(s) ds \right] dS \tag{50a, 50b}$$

and

$$Q = \int_{-\infty}^{\infty} f(s) \left[ \int_{-\infty}^{S} f(S) dS \right] ds = \int_{-\infty}^{\infty} f(S) \left[ \int_{S}^{\infty} f(s) ds \right] dS . \tag{51a, 51b}$$

Equation (49) gives the time dependent survival probability of a component exposed to stresses imposed randomly at an average rate of $\nu$ per unit time when the probability of surviving a single such load cycle is $R = 1-Q$. Combining Eq. (49) with Eqs. (3c) and (4a) the time-to-failure probability density and hazard rate functions are

$$f(t) = Q\nu e^{-Q\nu t} \tag{52}$$

and

$$\lambda(t) = Q\nu . \tag{53}$$

The case we are dealing with assumes that the static unreliability Q is constant. We see then that randomly stressing components of invariant strength does in fact correspond to the constant hazard rate, exponential time-to-failure pdf, exponential reliability situation.

Further insights can be developed from this model. If the stress environment is altered or if the part strength distribution is modified via design, materials, or manufacturing process changes, the static unreliability Q is changed to a new constant value. The exponential reliability model still applies but with a new value of the parameter $\lambda$. This is the basis for environmental, quality, derating, etc. factors employed in handbook prediction. If all applied stresses induce part failure, then $Q = 1$ and $\lambda = \nu$. The model continues to exhibit exponentially distributed times to failure and Poisson distributed failures per time interval. This is a well known relationship between these two distributions (See for example

Appendix 10.A of Ref. 3.). We can also see from Eq. (53) that the hazard rate $\lambda$ may be reduced by decreasing the stress/strength overlap unreliability Q. A plot of a typical stress/strength overlap situation is shown in Fig. 14. Again following Ref. 10 the unreliability Q is given via Eq. 51a as the area under the stress-at-failure distribution function

$$f'(s) = \frac{dQ(s)}{ds} = f(s)\left[\int_{-\infty}^{S} f(S)dS\right] = -\frac{dR(s)}{ds}. \qquad (54a, 54b, 54c)$$

There are a variety of situations for which the above description would not be adequate. If the component were required to operate in statistically distinct environments, Q would not be constant. Temperature dependence of the strength distribution would also be a complicating feature. If the part strength distribution is degraded by the application of stress, the unreliability Q would depend on loading history. The latter case represents a general class of wearout phenomena. We consider wearout in the next section of the report.

## 5.2.2    Wearout

Wearout refers to a systematic loss of functional integrity with time. This may be load or use induced as in the case of fatigue. Wearout may also proceed independently of loading as in the examples of corrosion or on-the-shelf degradation of unstable chemicals. The characteristics usually emphasized in connection with wearout phenomena are a strongly peaked time-to-failure pdf and its associated increasing hazard rate.

The stress/strength overlap description of the random hazard situation given in the previous section may be readily generalized to include wearout phenomena. In the simplest case we retain the feature of stresses Poisson distributed in equal time intervals. However, the stress/strength overlap unreliability Q (or per-loading-cycle probability of component failure) is taken to be time dependent. Thus $Q \to Q(t)$. If $Q(t)$ is a decreasing function, we are dealing with early failures or infant mortality. Wearout is, of course, described by an increasing $Q(t)$. The expressions for reliability, time-to-failure probability density function, and hazard rate analogous to Eqs. (49), (52), and (53) are

$$R(t) = \exp\left(-Q(t)\nu t\right), \qquad (55)$$

$$f(t) = \nu\left[Q(t) + t\left(\frac{dQ(t)}{dt}\right)\right]\exp\left(-Q(t)\nu t\right), \qquad (56)$$

and

$$\lambda(t) = \nu\left[Q(t) + t\left(\frac{dQ(t)}{dt}\right)\right]. \qquad (57)$$

In a wearout situation $Q(t)$ ranges monotonically from a low initial value to a maximum of unity. $Q(t) = 1$ represents such a severe strength degradation that

every applied stress may be expected to induce failure. This is a saturation situation [Q(t) cannot become any larger] so that dQ(t)/dt = 0. In this case of complete wearout we see that the hazard rate $\lambda$ becomes equal to $\nu$ the parameter of the Poisson distribution of applied stresses. From the close relationship of the Poisson and exponential distributions, $\nu$ is also recognized as the average rate of occurrence of the random applications of stress.

Equations (55) through (57) relate the important reliability/life functions to the stress/strength overlap parameter $Q(t)$ and the average frequency of stress occurrence $\nu$. In a still more general context the rate at which stress applications occur may itself be time dependent so that $\nu$ becomes $\nu(t)$. In addition the time dependence of the quantity $Q(t)$ may be due to both component strength degradation and time dependence of the distribution envelope of applied stress. The latter description applies to changes in service environment or conditions of use. Such a situation requires further generalization of Eqs. (55) through (57). This is streamlined by introducing a new parameter that contains all the time dependence of the problem. Let

$$z(t) = Q(t)\nu(t)t. \tag{58}$$

In terms of $z(t)$ the reliability, time-to-failure pdf, and hazard rate functions take the simple forms

$$R(t) = e^{-z(t)}, \tag{59}$$

$$f(t) = \left(\frac{dz(t)}{dt}\right)e^{-z(t)}, \tag{60}$$

and

$$\lambda(t) = \frac{dz(t)}{dt}. \tag{61}$$

We are now in a position to further compare and contrast life prediction and reliability prediction. In one sense the two approaches are totally equivalent via Eqs. (1a) and (3c). Usually, however, reliability prediction refers to drawing conclusions from actual time-to-failure experience with components or systems. This may be termed a macroscopic approach to evaluating the functional forms of $R(t)$, $f(t)$, and $\lambda(t)$ directly. Often this takes the form of using the available data to verify that some model such as the exponential, Weibull, or log normal is in fact appropriate. For the Weibull case, for example, one asserts $z(t) = \left(\frac{t-\gamma}{\eta}\right)^{\beta}$ and adjusts $\gamma$, $\beta$, and $\eta$ to best represent the data. In implementing the life prediction approach it would not be necessary to observe actual failures in normal field service. Rather one would characterize the operating stress environment (by ascertaining both a frequency profile $\nu(t)$ and an amplitude distribution $f(s)$ representing the loading situation). A study to determine the strength distribution $f(S)$ of the item of interest is also required. Complicating features are that $f(s)$ and $f(S)$ may themselves both be time dependent (perhaps implicitly via another factor such as temperature). If all of this information is obtainable, Eqs. (51) and (58) may be used to evaluate $z(t)$. The reliability, life, and hazard functions are then found via Eqs. (59), (60), and (61). Which of these two approaches is the more tractable one is a decision that must be made for each problem on its own merits.

A simplified wearout modeling exercise is presented in Section 5.4. Corrosion is taken to be the operative mechanism and the distributed character of important parameters is emphasized.

## 5.2.3    Mixed Populations

In reliability work it is common to generalize and classify causes of equipment malfunction as early, random, or wearout failures. Early failures are associated with hardware which was not delivered in satisfactory condition to begin with. Manufacturing defects or damage during inspection or shipment resulting in premature loss of function are examples of causes of early failures. A decreasing hazard rate, an initially very large time-to-failure pdf, and a sharply decreasing (initially) reliability function are associated with early failures. The random hazard situation has been discussed previously and refers to the chance occurrence of stresses large enough to induce failures in components of normal (ordinarily adequate) strength. For example a random failure of an automobile tire might be induced by impact with a foreign object on the roadway. Such an occurrence is random because there is nothing about it favoring one time interval over another (of equal duration). Random failures exhibit a constant hazard rate, exponential reliability attrition, and exponentially distributed times to failure. Wearout failures, on the other hand, are those that occur because component strength has eroded to the point of not being able to withstand the stresses of normal service. In this case the situation further deteriorates as time passes. The hazard rate is an increasing function, the time-to-failure pdf is peaked, and the reliability function is high at first and then falls sharply.

When a group of hardware items is subject to more than one of the above failure modes simultaneously it is termed a mixed population. For example a shipment may contain some initially defective as well as some normal units. This group would be expected to exhibit early as well as random or wearout failures. It is also not uncommon for random and wearout failures to be intermingled. An item can be characterized by a vulnerability to random overstress while processes are underway to erode the distribution of strengths from initial nominal values. In this case the same group of components would exhibit random and wearout behavior at the same time. More generally in a system certain components may show predominately random hazard behavior while others fail due to wearout. In either case a proper reliability/life description involves dealing with both aspects simultaneously. Early failures may have to be treated also although at the mature system level one prefers to have weeded out this category via some form of testing or burn-in procedure.

Formally dealing with mixed populations is straightforward enough although there are practical difficulties of course. The different aspects of the problem are taken to be independent so that the overall reliability is simply the product of the reliabilities of the relevant subclasses. If one proceeds from the life prediction point of view and these functions have been characterized, there is no problem. The situation is more complicated, if the reliability viewpoint is taken to interpret time-to-failure data. It must be recognized that no single familiar distributional model applies to this situation. In principle if the form of the superposition is known, curve fitting may be employed to fix the values of the relevant parameters. This approach is tenuous because the statis-

tical quality of reliability data is not usually sufficient to permit good separation of several factors contributing to the detailed description of a subtle superposition problem.

## 5.3  Implementation Philosophy

We have touched on the duality of reliability and service life concepts. One description implies the other. For the sake of establishing a consistent use of the nomenclature the following posture has been adopted in this report: Reliability prediction or the reliability viewpoint refers to interpreting actual equipment failures under actual use conditions. This would include time-to-failure data or numbers of failures in established intervals, for example. This is termed a macroscopic approach since it treats phenomenologically only observed failures. In contrast life prediction or the expected service life approach is a microscopic method that exmines in detail the conditions that cause failures to occur. Measurements are taken to establish a statistical description of the strength (in a generalized sense referring to resistance to a variety of types of stress) of the components of interest. The loading or stress environment must also be fully characterized.  This means determining the amplitude and time spectral features of applied loads.  In all but the most restrictive laboratory settings, this becomes a task of great complexity.  The method is therefore appealing when the reliability/life problem can be reduced to perhaps a single aspect of particular concern.

There is another area of activity usually called accelerated life testing that overlaps the two approaches discussed above.  Thus conclusions are drawn from time-to-failure data as in the reliability approach.  But this information is developed in a compressed time domain by manipulating (increasing the severity) of the applied stresses.  Interpreting accelerated testing requires a detailed correlation of the overstress situation utilized with the nominal stress conditions of primary interest.  This implies a simultaneous understanding of the problem from the stress/strength overlap viewpoint.  There is a substantial literature dealing with accelerated testing.  Chapter 9 of Ref. 11 is a good point of departure.  However, further discussion of this topic is outside the scope of this report.  As one can begin to see the reliability problem is staggering in scope.  Occasionally an impasse will be reached which can be resolved by failure analysis.  For example a mixed population situation may lead to data not described by a single model.  Examining the physical character of each failure may allow decomposition into subclasses that are more easily interpreted.

## 5.4  Dispersion Effects--An Example

In this section an example time-dependent stress/strength overlap calculation is presented.  Its discussion under this heading has to do with the important effects of distributional properties (in this case of the corrosion process taken to be inducing wearout).  The problem models the strength attrition of a cylindrical load bearing member under the influence of a corrosion process that decreases its radius at a constant average (but distributed) rate.  The process wearout endpoint or strength service limit is taken to be an arbitrary constant value.  This is equivalent to dealing with the situation where the reliability description is developed using a failure governing stress regarded as a deterministic (disper-

31

sionless) constant.  Adopting this approach allows the interesting reliability insights to be developed at a minimum cost in terms of computational complexity.

Our first concern in this corrosion modeling problem is to examine the dynamics of a migrating, spreading strength distribution function $f(S,t)$ sweeping across a defined endpoint $S'$ as shown in Fig. 15.  A general treatment of this leads to a formal expression for the time-to-failure probability density function.  The strength distribution is taken to be normal with time dependent mean and standard deviation.  Thus

$$f(S,t) = \left[\frac{1}{\sqrt{2\pi}\,\sigma_S(t)}\right] \exp\left[-\frac{1}{2}\left(\frac{S-\mu_S(t)}{\sigma_S(t)}\right)^2\right] . \qquad (62)$$

The unreliability of a single unit or the worn out fraction of a population of similar units equals the area $U(S',t)$ under the strength distribution to the left of $S'$.  Therefore, the worn out fraction is

$$U(S',t) = \int_{-\infty}^{S'} f(S,t)\,dS = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{S'-\mu_S(t)}{\sigma_S(t)}} e^{-\phi^2/2}\,d\phi \qquad (63)$$

where the change of variable $\phi = (S-\mu_S)/\sigma_S$ has been introduced.  Differentiation of $U(S',t)$ under the integral sign (which is simplified by the variable transformation that places all the explicit time dependence in the upper integration limit) yields an expression for the corrosion wearout time-to-failure probability density function $f_w(S',t)$.  This result is

$$f_w(S',t) = \frac{dU(S',t)}{dt} = \frac{1}{\sqrt{2\pi}}\left[\exp\left(-(\phi')^2/2\right)\right]\left(\frac{d\phi'}{dt}\right), \qquad (64)$$

where $\phi' = \phi(S')$.  Taking the indicated derivative of $\phi'$ we can express Eq. (64) in terms of the time dependent parameters $\mu_S(t)$ and $\sigma_S(t)$ of the strength distribution $f(S,t)$ directly as

$$f_w(S',t) = \left[\left(\frac{1}{\sqrt{2\pi}\,\sigma_S}\right)\exp\left(-\frac{1}{2}\left(\frac{S'-\mu_S}{\sigma_S}\right)^2\right)\right]\left[-\frac{d\mu_S}{dt} - \left(\frac{S'-\mu_S}{\sigma_S}\right)\frac{d\sigma_S}{dt}\right]. \qquad (65)$$

The required normalization $\int_{-\infty}^{\infty} f_w(S',t)\,dt = 1$ is apparent from inspection of Eq. (64).

To carry the corrosion modeling beyond the initial formal stages one needs to display the time dependence of the strength distribution explicitly.  This is done by distribution synthesis under the assumptions that a linear corrosion process operates to decrease the effective radius (and therefore load-bearing section) of an axially symmetric strength member.  The load bearing capability (or strength) is given by the product of the material tensile strength and the remaining sectional area.  Letting T represent the tensile strength and $r_o$, c, and t the initial radius, the corrosion rate, and time respectively; the strength function is

$$S = \pi T(r_o - ct)^2. \qquad (66)$$

32

For our present illustrative purpose $r_o$ and $c$ are taken to be normally distributed while T and t are regarded as deterministic parameters. Standard distribution synthesis arguments then lead to the desired expressions $\mu_S(t)$ and $\sigma_S(t)$ developed in Appendix D and displayed as Eqs. (67) and (68).

$$\mu_S(t) = \pi T\left((\mu_{r_o} - \mu_c t)^2 + \sigma_{r_o}^2 + \sigma_c^2 t^2\right) \tag{67}$$

$$\sigma_S(t) = \pi T\left(4(\mu_{r_o} - \mu_c t)^2(\sigma_{r_o}^2 + \sigma_c^2 t^2) + 2(\sigma_{r_o}^2 + \sigma_c^2 t^2)^2\right)^{\frac{1}{2}} \tag{68}$$

Taking T and t to be distributed would complicate these expressions but not particularly enhance the insights being developed in this modeling exercise.

At this point we have obtained a general expression for the corrosion wearout time-to-failure pdf and displayed explicitly the time dependence of the parameters appearing therein. The next steps are to take the required time derivatives, simplify the notation a bit by introducing auxiliary parameters, and calculate a representative group of numerical results for graphical display purposes. It is convenient to work in terms of the fractional strength $S$ defined as

$$S(t) \equiv \frac{S(t)}{\mu_S(0)} = \frac{S(t)}{\pi T\left(\mu_{r_o}^2 + \sigma_{r_o}^2\right)} , \tag{69}$$

which is distributed with parameters $\mu_S = \mu_S/\mu_S(0)$ and $\sigma_S = \sigma_S/\sigma_S(0)$. Some additional rescaling which complicates the notation slightly but simplifies the arithmetic is also implemented. A decomposition of the problem appropriate for numerical evaluation is included as Table IV. In Fig. 16 a typical time-to-failure distribution function is plotted as a function of the standardized time variable z (real time reexpressed on an initial mean radius divided by mean corrosion rate basis). The time-to-failure pdf is seen to be skewed to the right. Plotting the abscissa of this function on a logarithmic scale almost perfectly symmetrizes the distribution as shown in Fig. 17. Thus, the log normal distribution is an excellent representation of the results of this corrosion modeling exercise. The time-to-failure distribution was numerically integrated to form its cumulative. This allowed the corrosion wearout reliability function to be calculated for the model as unity minus the cumulative failure function. This function is compared in Fig. 18 with an exponential (random hazard) reliability function having the same MTBF. Additional graphs are presented representing the effects of different process endpoint choices ($S'$) and corrosion rate dispersions ($\sigma_c/\mu_c$) on the time-to-failure and reliability functions in Figs. 19 and 20. In the cases examined endpoint choice has a more pronounced effect than variability of the effective corrosion rate. One has to temper any conclusion drawn, however, with the observation that this is a Gedanken experiment and does not yet represent empirical inputs.

The corrosion modeling exercise has an immediate qualitative appeal for two reasons. The time-to-failure distribution is skewed to the right--exactly the result one expects of a process terminated as a spreading, symmetric distribution of strengths sweeps through a sharply defined endpoint. Also, the time-to-failure

33

pdf seems well represented by a log normal distribution as is often also the case for empirical corrosion studies. The assumptions leading to these results are quite simple--a crisp definition of end of useful life, initial part radii normally distributed, and clustered corrosion rates also taken to be properly represented by a normal distribution. For the examples considered one can summarize by noting that the time-to-failure distributions are strongly peaked and weakly skewed to the right. In a real experiment, structured along the lines of the Gedanken experiment just considered, exhibiting similar dispersion of the corrosion rate, and yielding limited time-to-failure stochastic data, one might be hard put to prefer a skewed description. This is an argument for paying close attention to the statistical design of reliability evaluation experiments.

## 5.5      Advantages, Limitations, and Difficulties

What we have chosen in this report to call life prediction is essentially simply the application of stress/strength overlap methods to hardware serviceability problems. With only the slightest change in viewpoint one would (perhaps more conventionally) call this probabilistic design for reliability. In any event, it is the microscopic approach that we have referred to involving a description in terms of the stresses operative in a given situation and the ability of hardware to function under specified loading conditions. The method is a very powerful and definitive one provided the detailed data requirements can be met. Full distributional information relating to load stresses and component strengths is needed. Clearly the scale of the problem for systems of even moderate complexity preempts the use of such a detailed approach. Life prediction can be most beneficially implemented when just one or a few areas of particular concern can be isolated. Even these situations will often require separate studies to characterize the failure governing stress and strength distributions. Hopefully this stiuation will improve if practicioners heed the appeal of Kececioglu and Cormier[10] to publish distributional data. Kapur[12] has observed that the problem can be simplified somewhat by concerning oneself only with the overlapping tails of typical stress/strength distributions. It is these regions that dominate the probabilistic unreliability.

In the sonar setting it may not be practical to attempt a full probabilistic description of the thermal, chemical, vibrational, and shock loading aspects of the exposed shipboard environment. Focusing more narrowly on suspected troublesome areas such as the processes that threaten housing integrity might prove both tractable and beneficial, however. A probabilistic approach has a particular appeal in that it represents a description emphasizing the distributional aspects of the reliability/life problem. Even when dealing with rather basic structural materials, Bondi[13] has pointed out that it is precisely the dispersion of their physical properties that strongly impacts their usefulness.

Probabilistic design methods have been under development on a rather broad front for twenty years or so. Very little of this pretentious structure has been displayed in this report. Thus the reader is cautioned not to underestimate either the labor or potential benefits of addressing sonar hardware problems probabilistically. Input information is the key. Given distributional data calculation of stress/strength overlap has been reduced to quadrature. Perhaps the most powerful numerical approach is Monte Carlo simulation. Computer programs for this have been developed.

## 6.0    RELIABILITY/LIFE DEMONSTRATION

Interest in product reliability is multifarious.  Common areas of concern
are defining realistic goals, upgrading an engineering design, reducing equipment
life-cycle costs, coping with state-of-the-art performance, and evaluating
deliverable equipment.  Reliability prediction is a tool in the hardware improvement
process.  But when it comes to obtaining the most definitive kind of statement
about the progress made, some form of demonstration test is required.  A general
but not too detailed discussion of several aspects of reliability/life evaluation
concerns follows.

### 6.1    Preferred Kinds of Information

Equipments can be evaluated in a number of ways.  But ordinarily the primary
concern is how well or for how long will the hardware meet mission performance
objectives under the conditions of use intended.  In one sense the most definitive
answer to this kind of question comes from accumulating actual field service
experience.  Often such information is unavailable and it is rarely timely for
making procurement decisions.  Next best with respect to determining the reliability
properties of interest is laboratory testing designed to substantially reproduce
field conditions.  Here timeliness may (or may not) be improved and questions of
cost have to be addressed for highly reliable equipment.  Applied stresses may be
increased to decrease test time or the number of units required to be set apart
for evaluation purposes.  This is accelerated testing and of course is itself not
without confounding features.  Thus full interpretation requires one to relate
real-time and accelerated test results and live with the uncertainties of the
description.  Against this background of complications some fairly efficient
means of evaluating performance and accepting or rejecting production lots have
been developed.  The methods are termed parametric or non-parametric depending on
whether or not a description is developed in terms of (the parameters of) a
characterizable distribution function.  Some examples are discussed in the
following sections.

Regardless of what approach best suits a particular application, from a
reliability point of view we will ask whether the item of interest is still
functional.  Is it failed or unfailed after some period of operation?  In
characterizing life we will need to know when the failure occurred relative to
when operation of the equipment began.  Obviously, then, it will be necessary to
decide specifically what excursions from nominal performance are to be considered
tolerable and which constitute failure.  If time-to-failure information is to be
acquired, monitoring procedures having an appropriate time resolution must be
implemented.  Some cautions are in order.  Test conditions must be similar to use
conditions if results are to be applied directly to equipment to be placed in
service.  Also it is important that test equipments be similar (meaning as closely
alike as manufacturing procedures allow) to hardware intended for field use to
allow valid inferences to be made.  To achieve the latter it is desirable to
implement some scheme to select an unbiased test sample from a larger homogeneous
production lot.

Summarizing the essential points, time-to-failure information is the most
useful type while total failures in an interval represents a class also of

considerable practical importance. The former are required for the construction of full distributional reliability/life descriptions. Total failure data permit the specification of a point reliability estimate (itself distributed as we have seen) but shed almost no light on time-to-failure distributions of more than one parameter. A point reliability description fleshed out in terms of producer and consumer risks is usually considered adequate for acceptance testing. A distributional description is to be preferred for failure mode diagnostics, prediction model development, and comparison with probabilistic design or physics-of-failure studies.

## 6.2      Inferring Distribution Parameters

Often one wishes to describe a reliability problem in terms of a mathematical model. The properties of a few important models were discussed earlier in the report. It is desirable that a particular model be advanced on the basis of physical arguments. Whether in practice a model is introduced systematically in this way or on a more pragmatic basis, a modeling exercise ultimately involves choosing the model parameters that best represent the available data. Several methods for this such as matching moments, probability paper plotting, or standard regression analysis are available. The latter two approaches are discussed further in the next two sections.

### 6.2.1      Plotting Methods

In establishing or making use of correlations between experimental data and the parameters of an associated physical model, it is often convenient to introduce variable transformations to linearize the model. The model is confirmed if the transformed data plot as a straight line on linear coordinate paper. Fitting the best straight line to the data can be done visually (avoiding numerical regression analysis). Furthermore model parameters can be inferred from the slope and intercept of the best-fit line making use of all the data at once. This gives roughly equivalent results and is more efficient than statistically processing the various point estimates separately.

Linear curve fitting can be further streamlined from the data analysts' point of view. To do this one builds the necessary mathematical rescaling directly into the coordinate axes of the graphical display. For example, consider the two-parameter exponential reliability model:

$$R = e^{-\lambda(t-\gamma)} . \tag{70}$$

Taking natural logarithms yields

$$\ln R = -\lambda t + \lambda \gamma . \tag{71}$$

Or equivalently

$$\ln (1/R) = \lambda t - \lambda \gamma . \tag{72}$$

Equations (71) and (72) are linear in the standard slope-intercept form $y = mx + b$ with independent variable t and dependent variable $\ln R$ or $\ln(1/R) = -\ln R$. Thus

lnR plots linearly on linear coordinate paper and R itself plots as a straight line on standard semi-logarithmic paper. There remain some provisos associated with the use of Eqs. (71) or (72). The reliability R is not a directly observable quantity but must be estimated on the basis of observed failures among similar equipments. We will digress to explore a preferred approach for this.

Reliability data requirements are discussed in Section 8.3. Anticipating the character of that discussion, imagine that we are blessed with a set of time-to-failure data to be analyzed. Times to failure may be grouped to generate a frequency histogram or ranked to synthesize a representation of the cumulative time-to-failure distribution of the population from which the sample was drawn. Johnson[14] has pointed out difficulties in inferring distributional properties using the former approach such as sensitivity to class interval choice when dealing with small samples. Further in a very pretty logical exposition[14] he has developed the median rank method of organizing ordered failure data. This approach has much to recommend it in connection with best characterizing the cummulative distribution representing the parent population.

Briefly paraphrasing some of Johnson's[14] introductory discussion we observe the following: Consider that a sample of N units has been selected (presumably randomly) from a larger population and tested to failure. The prerequisite for constructing a cumulative plot is to appropriately rank each failure. Thus, if the entire population were tested, each of the N subset failures would have a definite fraction of the population failing earlier. Correct specification of this quantity for a given observed failure would be its true rank within the overall failure distribution. Since the true rank is ordinarily unknown, the best we can do is estimate it. The estimate that has equal probabilities of being too high and too low is called the median rank. Johnson[14] shows that the true ranks of ordered failures within a subpopulation are beta distributed and that the median rank MR of the $j^{th}$ failure among N samples tested is obtained from the cumulative binomial distribution (partial binomial sum) via

$$\sum_{k=0}^{N-j} \frac{N!}{k!\,(n-k)!}\,(MR)^{N-k}\,(1-MR)^k \;=\; \frac{1}{2}, \qquad 0 < MR < 1 \cdot \qquad (73)$$

Equation (73) is of order N in MR but has uniquely one root in the interval 0 to 1. Tables of median ranks as well as other percentile ranks have been prepared by J. S. White and incorporated in Ref. 2. Some rank distributions and their median ranks are displayed in Fig. 21 for a sample of 10 units.

Let us return to the construction of probability plotting paper. Conventionally and for the reasons discussed some function of the cumulative time-to-failure distribution is plotted as a function of time, rescaled to yield a linear description. These plots are arranged to have positive slope. Consider the Weibull reliability function as a starting point for example

$$R \;=\; \exp\!\left[-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right] \cdot \qquad (74)$$

Taking reciprocals and then taking natural logarithms twice yields

$$\ln \ln (1/R) \;=\; \beta \ln(t-\gamma) - \beta \ln \eta. \qquad (75)$$

37

Or since $R = 1 - U$ and the median rank MR is a preferred estimate of the cumulative failure function U, Eq. (75) becomes

$$\ln \ln \left( \frac{1}{1 - MR} \right) = \beta \left( \ln(t - \gamma) \right) - \left( \beta \ln \eta \right). \tag{76}$$

Equation (76) is linear (in the rescaled quantities) in slope-intercept form. Weibull probability paper is constructed by building the required scaling into the coordinate labeling so that MR plotted versus $t - \gamma$ yields a straight line directly.

A variety of Weibull papers as well as probability papers for other distributions are obtainable. Ford Motor Company and General Motors have both developed Weibull papers for internal use. Probability plotting papers are commercially available from a company that identifies itself by the acronym TEAM (Technical and Engineering Aids for Management). A catalog of their special purpose graph papers is available on request. Contact

> TEAM
> P. O. Box 25
> Tamworth, N.H.  03886
> Telephone:  (603)323-8843.

More detailed descriptions of the use of probability paper are given in References 2, 15, and 16. The latter document discusses papers developed by R. A. Evans. Most probability papers label the ordinate axis as "percent failed" or "percent failure" while observed failure times are plotted as abscissas. The median rank has been discussed as a preferred estimate of the percent failed and thus can be used directly in probability plotting. Evans[16] recommends the essentially equivalent approach of plotting each datum twice at ordinates $r/n$ and $(r-1)/n$ where the notation refers to the $r^{th}$ ordered failure among n items tested. These two points fall on either side of the corresponding median rank and, of course, are easy to calculate. Both methods are applicable even if failures do not occur for all n items tested (censored test).

Some facsimile time-to-failure data including the specification of median ranks is displayed as Table V. This information is shown plotted on Weibull probability paper in Fig. 22. Three curves are shown representing different choices of the position parameter $\gamma$. Fixing $\gamma$ is an iterative procedure. If no position parameter can be found which linearizes (approximately--data are usually scattered) the Weibull plot, one concludes that the times-to-failure are not Weibull distributed. If a linear Weibull plot is obtained, the shape and scale parameters $\beta$ and $\eta$ are found via simple graphical procedures that vary slightly depending on the particular paper employed. Discussion of the uncertainties to be associated with the parameter values obtained via probability plotting is deferred to Section 6.3.

### 6.2.2    Curve Fitting

In the previous section we have considered inferring distribution parameters using probability plotting and visual curve fitting. This is a convenient and

widely accepted approach to standard regression analysis. When appropriate, regression analysis can be carried out with greater precision using numerical methods. We will explore this avenue here for utilitarian reasons as well as to develop further insights in the reliability/life context. Numerical regression analysis is commonly called curve fitting or least-squares curve fitting and is widely discussed in texts dealing with applied statistics. The specific approach to the subject that we will follow is developed by Bevington[17].

Suppose that we are dealing with an experimental situation where a dependent variable y is linearly related to an independent variable x via

$$y(x) = a_0 + b_0 x . \tag{77}$$

The quantities $a_0$ and $b_0$ are the true (but unknown) parameters of the linear model which we would like to estimate from a set of paired observations $(x_i, y_i)$. To make the example specific let us assume that very accurate observations of the $x_i$ are available while the $y_i$ are normally distributed with standard deviations $\sigma_i$ about the true (but unknown) values $y(x_i)$. The probability that the $i^{th}$ measurement will yield a value $y_i$ is then

$$P_i = \left( \frac{1}{\sqrt{2\pi}\, \sigma_i} \right) \exp\left[ -\frac{1}{2}\left( \frac{y_i - y(x_i)}{\sigma_i} \right)^2 \right] . \tag{78}$$

Provided the $y_i$ are independent the probability of making an entire set of N observations of $y_i$ at different $x_i$ is given by a product of N factors of the form of Eq. (78).

$$P(a_0, b_0) = \prod_{i=1}^{N} P_i . \tag{79}$$

We cannot actually evaluate Eq. (79) because we do not know the true linear model parameters $a_0$ and $b_0$. However, we can rewrite Eq. (77) in terms of estimates a and b of these quantities as

$$y(x) = a + bx . \tag{80}$$

Using Eq. (80) in Eqs. (78) and (79) yields the probability that the set of N observations is associated with the estimated values of the coefficients a and b. Thus

$$P(a,b) = \left[ \prod_{i=1}^{N} \left( \frac{1}{\sqrt{2\pi}\, \sigma_i} \right) \right]\left[ \exp\left( -\frac{1}{2} \sum_{i=1}^{N} \left( \frac{y_i - a - bx_i}{\sigma_i} \right)^2 \right) \right] . \tag{81}$$

The principle of maximum likelihood asserts that the set of measurements actually obtained experimentally is more likely to belong to the true parent distribution than to any similar distribution with different coefficients. Thus the best estimates of the parent parameters are obtained by maximizing the probability given in Eq. (81). This is accomplished by minimizing the argument of the exponential, or equivalently the function

$$\chi^2 \;=\; \sum_{i=1}^{N} \left[ \frac{1}{\sigma_i^2} \left( y_i - a - b x_i \right)^2 \right]. \tag{82}$$

Equation (82) can be minimized by requiring that its partial derivatives with respect to the parameters a and b simultaneously vanish.

$$\frac{\partial \chi^2}{\partial a} \;=\; -2 \sum_{i=1}^{N} \left[ \frac{1}{\sigma_i^2} \left( y_i - a - b x_i \right) \right] \;=\; 0 \tag{83a}$$

$$\frac{\partial \chi^2}{\partial b} \;=\; -2 \sum_{i=1}^{N} \left[ \frac{x_i}{\sigma_i^2} \left( y_i - a - b x_i \right) \right] \;=\; 0 \tag{83b}$$

Equations (83) are two linear, simultaneous equations in a and b which can be written more suggestively as

and

$$a \sum \left( \frac{1}{\sigma_i^2} \right) + b \sum \left( \frac{x_i}{\sigma_i^2} \right) \;=\; \sum \left( \frac{y_i}{\sigma_i^2} \right) \tag{84a}$$

$$a \sum \left( \frac{x_i}{\sigma_i^2} \right) + b \sum \left( \frac{x_i^2}{\sigma_i^2} \right) \;=\; \sum \left( \frac{x_i y_i}{\sigma_i^2} \right), \tag{84b}$$

where the sums are on i ranging from 1 to N. Equations (84) are readily solved using determinants yielding

and

$$a \;=\; \frac{1}{\Delta} \left[ \sum \left( \frac{x_i^2}{\sigma_i^2} \right) \sum \left( \frac{y_i}{\sigma_i^2} \right) - \sum \left( \frac{x_i}{\sigma_i^2} \right) \sum \left( \frac{x_i y_i}{\sigma_i^2} \right) \right] \tag{85a}$$

$$b \;=\; \frac{1}{\Delta} \left[ \sum \left( \frac{1}{\sigma_i^2} \right) \sum \left( \frac{x_i y_i}{\sigma_i^2} \right) - \sum \left( \frac{x_i}{\sigma_i^2} \right) \sum \left( \frac{y_i}{\sigma_i^2} \right) \right], \tag{85b}$$

where

$$\Delta \;=\; \sum \left( \frac{1}{\sigma_i^2} \right) \sum \left( \frac{x_i^2}{\sigma_i^2} \right) - \left[ \sum \left( \frac{x_i}{\sigma_i^2} \right) \right]^2. \tag{85c}$$

Equations (85) provide an analytical scheme for determining best estimates of the slope and intercept parameters of the experimental model given by Eq.(80). Formally one seeks the same kind of solution graphically using probability plotting methods. To comment on whether the two approaches are equivalent or not, one needs to examine both a little more closely. The assumptions we have made in constructing the least-squares method are:

1.  The $x_i$ values are dispersionless (without error).

40

2. Multiple values $y_i$ observed at the same $x_i$ are normally distributed.

3. The $y_i$ observed at different $x_i$ are independent (uncorrelated).

4. The regression line is centered on the mean of each normally distributed $y_i$.

5. The statistical weights $1/\sigma_i^2$ of each observation need to be specified.

6. The maximum likelihood concept is reasonable.

These statements are, in fact, appropriate in a variety of measurement situations. The approach can be made more general or more specific. For example if the $x_i$ are themselves distributed, their uncertainties can be reflected into the y-coordinates through the slope of the regression line and appropriately taken into account. A simplification often occurs if all data are taken using the same instrument in the same way. In this case, the standard deviations $\sigma_i$ may all be the same so that all data points are given the same statistical weight.

Against the properties of common forms of least-squares fitting let us look back at the nature of the probability plotting method discussed in Section 6.2.1. We might comment on each of the above assumptions individually:

1. In probability plotting the independent parameter is time directly or some function of time (such as lnt). It is treated as dispersionless although in any given experimental realization one must specify the precision of the time-to-failure information developed.

2. In probability plotting the ordinate values are typically $\ln(1-MR)^{-1}$ or $\ln\ln(1-MR)^{-1}$. As we have seen true ranks are beta distributed. As a practical matter the above functions may be roughly normally distributed but they are not expected to be rigorously normally distributed.

3. Median ranks are preferred estimators of order statistics in random samples from a uniform distribution. These order statistics are not independent (see, for example, Section 2.40 of Ref. 17). Thus the ordinate values used in probability plotting are not uncorrelated. This is probably the most serious obstacle to the realization of a straightforward and satisfying interpretation of time-to-failure data.

4. In probability plotting we are dealing with a visual fit to the data. We are not in a position to comment very specifically on how the ordinates are distributed or the regression line is positioned.

5. The median ranks used in probability plotting represent different beta distributions. A statistical weight reflecting the differing dispersions of different ordered failures should be constructed. No such adjustment is made in ordinary probability plotting. Another shortcoming is that the functional rescaling employed in constructing probability plots affects the parameter uncertainties as well

41

as the parameters themselves. Thus the same transformations should be used to reconstruct proper statistical weights.

6. Maximum likelihood ideas are felt to be very appropriate in a variety of settings. However, we have already flagged several anomalies between standard least-squares fitting and probability plotting. The departures seem to be sufficient to suggest that the adequacy of visual fitting be tested other than on the basis of any relationship to maximum likelihood.

We have called attention to a number of ways in which probability plots fail to rigorously meet the requirements associated with standard least-squares fitting schemes. Of course these objections are generally recognized and serve as the basis for placing probability plotting in proper perspective. Thus it is said that probability plotting does not have a definite statistical interpretation. The technique is recommended for rapid visualization of data trends. It should be used to discard models that are conspicuously inappropriate but not be relied on to select the best model from several apparently good ones. The Weibull model which is very interesting because of its versatility is also difficult to determine well via the probability plotting approach. That is, the parameter values obtained tend to carry large uncertainties.

Least-squares fitting schemes can be tailored to more directly deal with some of the features of the probability plotting problem. For example the need to rescale statistical weights can be obviated by fitting to a cumulative distribution directly rather than to a function linearized through coordinate transformations. Bevington discusses least-squares fitting to an arbitrary function in Chapter 11 of Ref. 17. Equation (82) is generalized to

$$\chi^2 = \sum_{i=1}^{N} \left[ \frac{1}{\sigma_i^2} \left( y_i - y(x_i) \right)^2 \right], \tag{86}$$

where $y(x)$ is an arbitrary function of x and a set of n parameters $a_j$. The function $y(x)$ may be expanded in a Taylor series. Retaining terms to first order in the parameter increments $\delta a_j$, Eq. (86) becomes

$$\chi^2 = \sum_{i=1}^{N} \left[ \frac{1}{\sigma_i^2} \left( y_i - y_o(x_i) - \sum_{j=1}^{n} \left( \frac{\partial y_o(x_i)}{\partial a_j} \delta a_j \right) \right)^2 \right], \tag{87}$$

where $y_o(x)$ is an initial estimate of the desired fitting function. Requiring as before that the derivatives of Eq. (87) with respect to $\delta a_j$ simultaneously vanish yields for $k = 1, \ldots, n$

$$\sum_{i=1}^{N} \left[ \frac{1}{\sigma_i^2} \left( y_i - y_o(x_i) - \sum_{j=1}^{n} \left( \frac{\partial y_o(x_i)}{\partial a_j} \delta a_j \right) \right) \frac{\partial y_o(x_i)}{\partial a_k} \right] = 0. \tag{88}$$

Equations (88) are a set of n simultaneous, linear equations in $\delta a_j$, the corrections to the initial parameter estimates. Since Eqs. (88) are only asymptotically correct as $y_o(x)$ approaches the true regression profile, iteration

42

is required. Equations (88) are solved using determinantal or matrix inversion
methods. The resulting $\delta a_j$ are used to construct an improved test function
$y_o(x, a_j) \to y_o(x, a_j + \delta a_j)$. The process is repeated until $\chi^2$ reaches a stable
minimum. Convergence is rapid usually requiring only three to five iterations.

A full blown iterative least-squares fitting program for matching the
Weibull cumulative distribution to median rank versus time-to-failure data is
presented as Appendix E. The program is written taking the a priori statistical
weights $1/\sigma_i^2 = 1$. It includes calculation of the uncertainties to be associ-
ated with the final fitting parameters based on the error analysis developed by
Bevington[17]. Although the fitting program is set up to treat the Weibull
cumulative distribution, its use is much less restricted. The reader may use
Appendix E for other curve fitting problems simply by substituting another
fitting function and its first derivatives with respect to each of the parameters
involved.

Unfortunately improved curve fitting does not solve all the problems
associated with interpreting time-to-failure information. Ascertaining the
parameters of time-to-failure distributions raises general questions in the
theory of order statistics and is discussed in greater detail in Chapter 5 of
Ref. 11. This source refers in particular to an impressive series of papers by
N. R. Mann dealing largely with the Weibull model.

## 6.3 Quality of Description

The plotting and curve fitting methods just discussed relate to the question
of estimating the parameters of cumulative time-to-failure distributions that
best represent observed, ordered time-to-failure data. When these results are
obtained, one necessarily inquires about their quality or dispersion. A couple
of ways of addressing this kind of question are discussed in the next two
subsections of the report. In reliability studies the conclusions one draws
from this are usually not very satisfying owing in part to limited data but
mostly to the distributional aspects of the problem. Thus the parameters of
interest exhibit substantial uncertainties. This has nothing to do with the
power or efficiency of regression analysis per se. Nevertheless it is a
frustration to those interested in characterizing hardware reliability.

### 6.3.1 Dispersion Estimates

Evans[16] in connection with the use of probability papers gives graphical
constructions for estimating the errors to be associated with linearized visual
regression analysis parameters. Actually his description incorporates results
of one of the statistical goodness-of-fit tests discussed in Section 6.3.2.
Bevington[17] discusses the estimation of uncertainties associated with parameters
obtained by curve fitting. These are of course related to the overall "goodness-
of-fit" obtained in the analysis. But there is a distinction between error
analysis and goodness-of-fit tests. The former measures the dispersion of
parameters obtained by curve fitting. The latter measure the probability that
the observed data in fact belong to the distribution tested (with its parameters
specified).

43

In this section we pursue the error analysis line of reasoning briefly. As Bevington[17] points out the error $\sigma_{a_j}$ associated with a fitting parameter $a_j$ is due to the experimental errors $\sigma_i$ collectively, according to the weighted sum

$$\sigma_{a_j}^2 = \sum_{i=1}^{N} \left[ \sigma_i^2 \left( \frac{\partial a_j}{\partial y_i} \right)^2 \right], \tag{89}$$

where $y_i$ is the $i^{th}$ datum of N total. Using this result the squared uncertainties associated with Eqs. (85a) and (85b) (two-parameter linear regression) are

$$\sigma_a^2 = \frac{1}{\Delta} \sum \left( x_i^2 / \sigma_i^2 \right) \tag{90a}$$

and

$$\sigma_b^2 = \frac{1}{\Delta} \sum \left( 1/\sigma_i^2 \right) , \tag{90b}$$

where $\Delta$ is given by Eq. (85c). The method is readily generalized to higher order linear regression models and to curve fitting to an arbitrary function. Bevington[17] constructs the logical extension to the latter case by considering the change in a single parameter which produces a unit increase in the least squares fitting statistic $\chi^2$ [Eq. (86)] minimized with respect to the other parameters. All of these cases are then neatly described by expressing parameter uncertainties in terms of the error matrix $\varepsilon$ via

$$\sigma_{a_j}^2 = \varepsilon_{jj}. \tag{91}$$

The error matrix is defined as the inverse of the curvature matrix (in fitting parameter hyperspace). Thus

$$\varepsilon_{jk} = (\alpha^{-1})_{jk} , \tag{92}$$

where the elements of $\alpha$ are given by

$$\alpha_{jk} = \sum_{i=1}^{N} \left[ \frac{1}{\sigma_i^2} \left( \frac{\partial y(x_i)}{\partial a_j} \right) \left( \frac{\partial y(x_i)}{\partial a_k} \right) \right] . \tag{93}$$

If the data point uncertainties $\sigma_i$ are unknown, they can be estimated from the overall data record itself via

$$\sigma_i^2 \simeq \frac{1}{N-n} \sum_{i=1}^{N} \left( y_i - y(x_i) \right)^2 . \tag{94}$$

Equations (91) through (94) are used in Appendix E to estimate the regression parameter uncertainties.


## 6.3.2  Goodness-of-Fit Tests

Goodness-of-fit tests are structured to provide some measure of the likelihood that a given set of observations (sample test results) in fact belong

to some specified distribution. The distribution parameters may be given a priori or obtained from analyzing the data record itself. Two of the better known goodness-of-fit tests are discussed in this section. These are the test developed by Pearson[18] and the Kolmogoroff-Smirnov test. References 11, 19, and 20 all provide chapters on goodness-of-fit or statistical inference tests with the latter two being most appropriate for our present purpose. All of these sources identify additional reference material. Kececioglu[21] gives a particularly lucid description of the practical application of these two methods.

The $\chi^2$ test is based on the proposition that the statistic

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{95}$$

is approximately $\chi^2$ distributed with $\nu = k-m-1$ degrees of freedom where

- $k$ = number of class intervals into which the data are grouped

- $m$ = number of parameters of the test distribution obtained from the data record itself

- $O_i$ = observed event frequency in the $i^{th}$ class interval

- $E_i$ = theoretically expected event frequency in the $i^{th}$ class interval.

Roughly speaking the preferred number of classes is 5, 7, or 9 depending on whether the total number of observations is of order 10, 100, or 1000 respectively. One also prefers that each class interval contain at least 5 events. Class intervals need not all be the same size in event parameter space (time, cycles to failure, etc.) to accomplish the latter. A number of examples of grouping data and setting up the test are provided in the references cited above. Ultimately one calculates the test statistic according to Eq. (95) and compares it to tabulated percentile values of the chi-square distribution. One can write the probability statement

$$1 - \alpha = P\left( \chi^2 \le \chi^2_{(1-\alpha),\nu} \right) = \int_0^{\chi^2_{(1-\alpha),\nu}} f(\chi^2)d\chi^2. \tag{96}$$

Equation (96) states that if the inequality (equality) is satisfied, the original test hypothesis is confirmed at the $\alpha$ level of significance. For example if $\alpha = 0.05$ and $\nu = 4$, there is only a 5% probability that the test statistic $\chi^2$ will exceed the critical value $\chi^2_c = \chi^2_{0.95,4} = 9.49$ [note some tables give $\chi^2_{\alpha,\nu}$ rather than $\chi^2_{(1-\alpha),\nu}$] for a given set of observations for which the initial distributional hypothesis is correct. Occurrence of the outcome $\chi^2 > \chi^2_c$ is considered unlikely (at the specified risk or significance level $\alpha$) and is therefore the basis for rejecting the original hypothesis.

As an example let us test the proposition that the time-to-failure data of Table V belong to a two-parameter Weibull distribution with parameters $\beta = 2.3$

and $\eta$ = 10,000 hrs as obtained from Fig. 22. Implementation of the $\chi^2$ test is shown in Table VI. Notice that we have contrived to have 5 events per class interval by choosing unequal class intervals. Having only 20 data points to work with is still a little confining and allows only 4 classes in connection with the former choice. As we see from Table VI the two-parameter Weibull model is not rejected at the 5% significance level.

The reader is referred to the literature for additional operational level information associated with implementation of the $\chi^2$ test. To summarize, the essential elements of the method are:

1. Select the distribution (pdf) to be tested.

2. Choose the desired level of significance $\alpha$.

3. Specify the parameters of the distributional hypothesis (perhaps by fitting the data itself).

4. Decompose event space into class intervals.

5. Tally the observed data by class to obtain the observed frequencies.

6. Calculate the expected class frequencies (by taking differences of the cumulative of the test distribution evaluated at the class boundaries).

7. Form the test statistic $\chi^2$ [see Eq. (95)].

8. Calculate the number of degrees of freedom for the problem.

9. Compare the test statistic with the critical value obtained from tables of the $\chi^2$-distribution.

The Kolmogoroff-Smirnov test statistic d is the maximum absolute difference of two cumulative distribution functions for some observed set of values of the independent variable x. Thus for n observations

$$d = \max \left| S_n(x_r) - F(x_r) \right|, \tag{97}$$

where

$S_n(x_r)$ = observed cdf at $r^{th}$ failure

$F(x_r)$ = hypothesized cdf at $x = x_r$.

Equation (97) is asymtotically distributed as[22]

$$\lim_{n \to \infty} P\left(d > C/\sqrt{n}\right) = 2 \sum_{m=1}^{\infty} (-1)^{m-1} \exp\left(-2m^2 C^2\right). \tag{98}$$

This result together with exact calculations of the probability $P(d > C/\sqrt{n})$ for small n allow tables of critical values of the Kolmogoroff-Smirnov test statistic

to be constructed.[23,24] This information is now also commonly reproduced in textbooks treating statistical inference and statistical methods for reliability. In evaluating Eq. (97) the observed cumulative distribution at the $r^{th}$ failure is estimated by the rank fraction

$$S_n(x_r) = r/n .$$ 
(99)

Or if the data are organized into m groups (rather than n groups of 1), Eq. (99) generalizes to

$$S_n(x_{r_g}) = r_g/n ,$$
(100)

where the m quantities $r_g$ are the failure order numbers corresponding to the upper group boundaries. As was done for the $x^2$ test the Kolmogoroff-Smirnov test statistic is compared with tabulated critical values corresponding to some stated level of significance. Application of the test again to the data of Table V is displayed in Table VII. We conclude as before that the two-parameter Weibull model ($\beta = 2.3$, $\eta = 10,000$ hrs) cannot be rejected.

Again we might summarize the basic features of the Kolmogoroff-Smirnov goodness-of-fit test.

1. Select the cumulative pdf to be tested.

2. Choose the desired significance level.

3. Specify the parameters of the distributional hypothesis (preferably not by fitting the observed data).

4. Tabulate the observed data by rank fraction to obtain the experimental cumulative distribution.

5. Calculate the corresponding expected cumulative distribution values for the test hypothesis.

6. Take differences of (4.) and (5.) and identify the Kolmogoroff-Smirnov test statistic.

7. Compare the test statistic with tabulated values and draw a conclusion.

Some provisos associated with the use of the two goodness-of-fit tests described in this section of the report are:

1. The $x^2$ test is preferred for evaluating discrete distributions while for continuous distributions one should favor the Kolmogoroff-Smirnov test.

2. The $x^2$ test is suitable for situations where the values of parameters used in specifying the test hypothesis are obtained from the same data record as is used in the test itself.

3. The conditions of item (2.) compromise the Kolmogoroff-Smirnov test.

4.  The data grouping required in the $\chi^2$ test precludes its use with very small samples.

5.  There are no restrictions on applying the Kolmogoroff-Smirnov test in small sample situations.


## 6.4      Testing--Context and Cost

Much of this report thus far has related to structuring the analytical machinery for interpreting reliability/life information. Testing is simply the systematic exercising of equipment to yield failures from which reliability inferences can be drawn. Thus given copious amounts of this kind of information we can construct models and evaluate their parameters. Often this takes the form of trying to ascertain whether contractural obligations are being met in connection with a particular procurement. This latter kind of evaluation is called acceptance testing. In acceptance testing one is concerned with the tradeoff problem of being fair to both consumer and producer while at the same time being reasonably precise about discriminating between superior and inferior equipment. The situation is usually quantified in terms of the producer's risk $\alpha$, the consumer's risk $\beta$, and the discrimination ratio k. The producer's risk is the probability that equipment of adequate quality will be rejected by the test. The consumer's risk is the chance the buyer takes that actually inferior hardware will be judged acceptable. The discrimination ratio is the quotient of the nominal or upper level of desired performance (MTBF for example) and the minimum acceptable or lower performance level. These quantities are identified in Fig. 23 which is one form of the operating characteristic (OC) curve which shows for a particular underlying distribution the probability of passing an acceptance test versus the true performance attribute of the equipment being evaluated. The detailed shape of the OC curve depends on the sample size and the level of performance demanded. Reduced consumer and producer risks and higher discrimination (smaller k) require more testing.

In general the statistical interpretation of reliability tests can become quite involved. One needs to consider whether the test is time terminated, failure terminated, censored, or uncensored and whether the data obtained are time-to-failure, failures per interval, or total failures in total time information. Does one know in advance from what distribution the sample is drawn or is this to be inferred from the test? Our purpose is not to explore all of these avenues here but rather to focus on certain economies that have been developed. A great deal of modern acceptance testing is based on the pioneering work of Abraham Wald[25] in the area of sequential testing. In this case for a specified underlying distribution one establishes an open ended test plan and keeps track of a probability ratio statistic relating to the probabilities that the observed number of failures belong to a realization of the upper and lower performance limits. The accept/reject decision is based on the behavior of this statistic and such a test is called a probability ratio sequential test or simply a sequential test. Sequential testing is most fully developed for the exponential case. A variety of test plans, operating characteristics, and expected test time characteristics for this situation are displayed in Ref. 26. A typical format for sequential testing sampling plans or decision making plots is that shown in Fig. 24. Cumulative failures are plotted

versus cumulative equipment operating time. Migration of the stepwise plot line outside the "continue testing" region results in an accept or reject decision being reached. The truncation boundaries $t_o$ and $r_o$ are due to Epstein.[27]

Additional details concerning the design of a sequential test plan tailored to a particular application are presented in Ref. 2. We do not elaborate on this here because we are ultimately more interested in making actual hardware reliability improvements than fine tuning the evaluation process for the exponential model. Sequential testing has much to recommend it, however. Wald[25] has shown that a sequential plan has an average risk no greater than a test where the sample size is chosen in advance. On the other hand good units are promptly accepted while bad units are rejected efficiently in terms of the test time required to make a decision. Reduced time and expense associated with testing is a principle advantage of the sequential approach. Not surprisingly (though perhaps unfortunately) the greatest test time required is associated with coming to a decision when the true reliability is close to the design objectives (upper and lower test limits). While the economies of sequential testing are real, the amount of time that must be invested in hardware evaluation is still significant. Inspection of the test plans of Ref. 26 shows that the cumulative test time required ranges typically from 2 to 20 times the MTBF value being demonstrated.

In the case of routine acceptance testing one often has well developed expectations concerning how the test should turn out. This may be based on experience with similar equipments previously evaluated. In such cases it is possible to realize an additional reduction in required test time by using the celebrated and controversial methods of Bayesian inference. This subject area has developed around the conditional probability theorem first established by Bayes[28] over two hundred years ago. For the reader to whom Bayesian inference is new, Ref. 29 is a suggested point of departure. Reference 29 is a special issue of "IEEE Transactions on Reliability" devoted exclusively to Bayesian inference. Most of the papers have a review orientation. One of the advertised benefits of Bayesian theory is that it allows subjective or personal preference kinds of inputs. There is continuing dialogue concerning whether this is permissible in science, how it should be done, and what Bayesian forms are appropriate in treating reliability problems.

Following Refs. 11 and 30 Bayes theorem may be stated as

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)} , \qquad (101)$$

where in the reliability context the elements of Eq. (101) have the following interpretations:

$A_i$        a set of mutually exclusive and exhaustive (for B) hypotheses or belief statements

$B$        an event or piece of evidence that relates to the truth or credibility of the $A_i$

49

$P(A_i)$    elements of the prior probability distribution, that is, the
probabilities assigned to the hypotheses $A_i$ before evidence B
becomes available

$P(B|A_i)$   likelihoods or conditional probabilities that the evidence B
will obtain assuming the truth of each of the $A_i$ separately

$P(A_i|B)$   posterior probabilities of the $A_i$ given the evidence B .

The denominator of the right side of Eq. (101) is the total probability of the
evidence B calculated by weighting the $P(B|A_i)$ by the hypothesis probabilities
over the entire ensemble. That Eq. (101) is a correct logical statement is not
disputed. However, if unrealistic prior information is supplied, conclusions
drawn from using Eq. (101) may be expected also to be unrealistic and of little
value. This is the center of the multifaceted Bayesian controversy. Is
mathematical convenience sufficient justification to prefer conjugate forms of
the theory (prior and posterior distributions belonging to the same functional
family)? Are prior distributions unsupported by actual data to be considered
legitimate? Should one prefer continuous or discrete descriptions? There are
other pitfalls to the uninitiated. Some forms of the theory emphasize what are
called loss and risk functions (see Refs. 20 and 21 for example). In this
approach one seems to be more concerned with the impact of his decisions than
their empirical basis.

It is not our purpose here to present or elaborate on Bayesian inference
theory in any detail (This has been the subject of a number of books and very
many technical papers.). We simply wish to call attention to its existence and
its apparent relevance to reliability problems. Perhaps some additional guidance
and accession to the literature can be provided as well. Reference 31 considers
sequential testing from a Bayesian viewpoint and shows sampling plans quite
suggestive of Fig. 24 of this report. Reference 32 discusses obtaining prior
distributions from available data for actual hardware equipments (mostly
electronic). Reference 30 presents a very appealing demonstration of the
advantages of a discrete Bayesian formulation and the practicalities of its use
in treating reliability problems. Intriguingly this source suggests that viable
Bayesian prior distributions be arrived at by committee in what amounts to an
engineering design review setting. A direct comparison of fixed sample size,
sequential, and Bayesian reliability demonstration testing plans with respect
to their relative efficiency in terms of required test time is given in Ref.
33. Obviously when it can be properly structured, Bayesian inference is very
efficient.

As has been mentioned this report is more concerned with improving the
design of sonar transducers than evaluating current production. Nevertheless
design improvement begins by trying to keep what is right about the item in
question and change what is wrong. With respect to both of these categories in
the sonar setting there seems to be a wealth of information which one might
like to process using Bayesian methods. The scope of this report does not
allow the development of solutions of this kind here. Only encouragement to
carry on can be provided.

## 7.0    SPECIAL RELIABILITY DIFFICULTIES FOR NAVY SONAR EQUIPMENT

Thus far in this report we have developed a number of topics that are a part of the standard machinery for dealing with reliability problems. The area of wet-end sonar equipment offers some unique challenges in applying these methods as we shall see in this section.

### 7.1    Heroic Time Scale

Numbers like 100,000 hours have been written into recent sonar transducer procurements as the required mean-time-between-failures statistics (MTBF's) for these equipments. I am not suggesting that such a performance objective is unrealistic. Earlier it was pointed out that sonar transducers are rather uncomplicated and long life should be realizable on the basis of their structural simplicity (and sound design). Nevertheless the MTBF's called for must be recognized as large numbers. By way of comparison the subsystem MTBF's of modern jet fighter aircraft range from a few hours to tens or hundreds of hours.[34] Taking these elements together the entire aircraft may exhibit an MTBF in the range 0.5 to 3 hours.[34] The sonar transducer reliability statistic is seen to be 5 orders of magnitude larger than this. It becomes unrealistic to construct a conventional acceptance test to assure the Navy that it is receiving what it bargained for. In the limited procurement setting such a test would be prohibitively expensive and the results would not be timely. New approaches to characterizing the reliability of long lived systems should be cultivated. This might take the form of demonstrating reliability after the fact through fleet experience and contriving to achieve it in future procurements through controlled engineering practices.

### 7.2    Gaps in the Quality and Kind of Hazard Rate Data

Contractors bidding on sonar transducer procurements are usually asked to prepare a handbook-style prediction of the reliability of the item in question. This ordinarily requires quite a bit of creativity since the standard data sources such as Ref. 7 do not provide the necessary information. Nevertheless the task is invariably completed and a predicted reliability slightly superior to that requested by the Navy is advertised. (One could hardly do otherwise and expect to win the contract.) All aspects of this exercise—what the Navy asks for and what the contractors deliver—seem a bit misdirected. Structural arguments have been presented that one would look for wearout phenomena rather than the exponential reliability modeled. Fleet service data[35] acquired on TR-155F transducers incorporated in the AN/BQQ-5 sonar system confirm this. These data represent a characteristically wearout-like cumulative time-to-failure function. In this case the life-limiting process is identified as corrosion and debonding along the rubber-window/headmass-shroud interface. Our thesis in this section is that a basis for exponential prediction modeling of sonar transducers does not in general exist. Further there is presently insufficient information available to engage in any detailed predictive modeling of a new design that is significantly altered from its predecessors. The call to action in this is that the sonar community itself must develop its own relevant data sources and reliability experience.

## 7.3 Need to Define Systems Operating Requirements

In Section 5.1 we foreshadowed the need for precise definitions of the performance requirements of hardware of interest. For example in the sonar case the reliability required of a transducer will depend on the array configuration of similar units and the desired array performance. Some studies have been carried out by sonar systems personnel relating to the degradation of array beam forming characteristics as a function of elements lost to service. One needs further to relate this kind of analysis to characteristics of ultimate interest such as target recognition capability. It is only when the operating characteristics of the system are defined and related to array parameters that the reliability specialist can construct a specification for a single transducer coordinated with overall mission objectives. It is often taken as a general rule that an array must be 90% intact to function adequately. To improve on this description requires closer cooperation between sonar systems and reliability personnel than has heretofore been practiced.

## 7.4 Undefined Process Endpoints

In order to avoid wearout failures it is desirable to employ preventive maintenance as a tactic. Often this involves monitoring some component attribute and replacing the part when a specified service limit is reached. Alternatively a regular replacement interval may be established without regard to evaluating the apparent condition of the item involved. In the sonar transducer case there are identified wearout processes for which a service limit has not been specified. For example, how much corrosion of the housing can be tolerated before the risk of perforation is considered unacceptably high? Or a much more tantalizing illustration is the following: It is felt that it is undesirable to have water inside a transducer due to its role in promoting corrosion and degrading electrical breakdown characteristics. Elastomers, particularly neoprene and polyurethane rubbers, are often used as the primary moisture barriers in projector and hydrophone installations. But it is known that these materials are permeable to moisture.[36,37] Thus the important question is not whether water is present in transducers but how much can be tolerated. Dessicants are often incorporated to reduce transducer humidity levels. But the basic question together with what its life-limiting implications are remains unanswered. Moisture is a concern in both gas-filled and oil-filled transducers. In the latter case the solubility of water is an important fill fluid characteristic. There have been transducer designs of both the gas-filled and oil-filled types that have given good service. Still a full understanding of why some designs outperform others seems to await research on the role water plays in fostering wearout processes.

## 7.5 Test Method Nonuniformity

Some of the complicating features already mentioned in connection with sonar applications have led to differing responses within the community. Thus a variety of ways of evaluating equipment have evolved. Before a transducer is mounted on a ship, definitive acoustical evaluation can be carried out at any of several specialized Naval facilities. After transducers are mounted some

52

array evaluation work can be performed using prepared targets. For the most part, however, simpler schemes for testing transducers are preferred. Thus most transducer diagnostic activity involves making simple resistance measurements at inboard terminal boxes servicing the transducer electrical cables. To what degree these resistances correlate with the acoustical performance of corresponding units is not established. The test is not designed to distinguish between cable difficulties and anomalous behavior of the transducer itself. In some systems there is a disconnect criterion at which point an individual transducer is no longer felt to be beneficially contributing to the overall array performance. Thus when the resistance of a given transducer falls below this critical value, the unit is electrically removed from service. There is generally another service limit on transducer resistance which calls for replacement during an overhaul. Usually this replacement boundary represents substantial deterioration of the resistance specified for new equipment.

Several aspects of this situation are somewhat disquieting from a reliability evaluation point of view. Different sorts of test entirely (acoustical versus resistance) are associated with the qualification of new equipment and its ultimate removal from service. The disconnect criterion probably has a basis in signal processing theory. On the other hand the replacement resistance criterion seems arbitrary and but little related to acoustical performance.

## 7.6    Need for Systematic and Uniform Data Acquisition

For a variety of reasons stated above reliability prediction associated with sonar transducer procurements is very difficult. The time scale is heroic, production is often quite limited, and operating requirements and life-limiting processes are frequently incompletely characterized. Thus until some of the basic open questions relating to permeation effects, bond degradation, corrosion, electrical breakdown, etc. are answered, it seems unrealistic to expect to realize specific reliability objectives in connection with any given new procurement. However, all transducer systems are monitored and maintained. Thus by focusing attention on transducers in place in the fleet, one ought to be able to identify problem areas and suggest improvements to be implemented on later models. We have already seen that limited testing during hardware design can also flag overt problems and lead to much improved equipment. In the latter case the benefits are more immediate. In both cases the equipment may be better but the thrust of prediction is frustrated in that one does not know how good. In the sonar setting it seems most realistic to demand an answer to this question only after fleet experience is acquired. If this experience is satisfactory, build new units in the same way. If improved performance is desired, identify the problem areas and make changes.

The basis for progress along the lines just discussed is eternal vigilance. That is the performance of sonar systems should be systematically monitored and complete records kept. Care should be taken that tests are uniformly applied. For the purpose, resistance measurements are certainly admissible whether or not correlations with acoustical performance are established. It is recognized that sonar transducer diagnostics can be developed only on a limited opportunity basis. This in itself is not a fundamental problem but does serve to punctuate the need for keeping good records and treating each maintenance

53

interval as an opportunity to acquire reliability data. To the procurement manager seeking near term results, this program may seem inadequate. No superior alternative suggests itself although parallel efforts on the development of accelerated test methods is probably worthwhile. One should remember that the present state of the reliability improvement art for sonar transducers is well described as stumbling from crisis to crisis. It is only by recognizing where one is beginning that plans for progress can be well structured.

## 8.0 RECOMMENDATIONS

Thus far in this report we have tried to identify the nature of the sonar transducer reliability problem. In addition some of the relevant analytical machinery for dealing with reliability has been introduced. Now we will treat briefly some of the ways in which the latter may be correctly applied to the former. Some technical and some operational concerns will be considered.

## 8.1 Methods Applicability

As has been observed a description of sonar transducer reliability has some unique features. At least some wearout processes occur in fleet service and in certain cases tend to dominate the reliability description. Thus for the most part handbook prediction methods of either the Part Stress Analysis or Parts Count type seem to be inappropriate. Of course a formal separation into component groups which may be exponentially reliable or exhibit wearout can always be made. The overall reliability is the product of the subclass reliabilities. Failure rate experience needed to support such an approach, when available at all, is likely to be dispersed among various production contractors rather than centrally cataloged.

The probabilistic design approach discussed in Section 4.2.2 is generally applicable in principle. It is a microscopic description enabling one to characterize reliability on a per-stress-application basis. The connection between probabilistic design and what we have called macroscopic reliability was dealt with in Sections 5.2.1 and 5.2.2. Probabilistic design requires a distributional-level description of environmental and service stresses and component strengths. When this information is available, the method is very powerful. However, acquiring the information for a particular problem area of interest may necessitate a separate research program.

We see that in some ways the nature of the sonar reliability problem has a pejorative impact on efforts to apply standard methods. In other ways the situation is ameliorating. The distribution of the hardware of interest to us is limited to a single customer--the Navy. This customer is highly organized and meticulous in dealing with maintenance and renewal functions. The inventory control process is itself practically a macroscopic reliability experiment.

The Navy sonar setting also exhibits a rather controlled evolutionary characteristic. New procurements are typically only slightly altered from the generation of hardware being supplanted. Many features such as ceramic configurations, prestressing arrangements, and coupling and decoupling provisions are pretty well established. In this setting of slow change Bayesian inference methods would seem to be particularly appropriate.

In this section it is our purpose to evoke neither optimism nor gloom. We simply wish to point out that a reliability program can be no stronger than the true overlap of the methods employed with the problem addressed. Obviously such a program must be structured by individuals capable of exercising the necessary critical judgment in the somewhat esoteric reliability arena.

## 8.2    Recognizing the Statistical Character of the Problem

The title of this section is somewhat curious in a report for which the statistical aspects of reliability have represented a major focus throughout. But one significant feature has not yet been emphasized--the question of product similarity. We have seen that reliability cannot be related macroscopically to the failure of a single item. Rather inferences must be drawn from the behavior of a population of similar items. Ah but given such a situation, can we tell whether the units are similar (or nearly identical) or not. Let us look into this. Suppose that a number of items of the same kind are in fact exponentially reliable (an assumption not requiring justification for the moment) but exhibit different constant hazard rates. Take these hazard rates $\lambda$ to be normally distributed as

$$f_N(\lambda) = \left(\frac{1}{\sqrt{2\pi}\,\sigma_\lambda}\right) \exp\left[-\frac{1}{2}\left(\frac{\lambda - \mu_\lambda}{\sigma_\lambda}\right)^2\right]. \tag{102}$$

Then since the reliability of an individual component is $R(t) = e^{-\lambda t}$ and Eq. (102) is a normalized pdf, the reliability of the population is found by weighting the constituent reliabilities according to

$$\overline{R(t)} = \int_{-\infty}^{\infty} f_N(\lambda)\, e^{-\lambda t}\, d\lambda. \tag{103}$$

Equation (103) is easily evaluated yielding

$$\overline{R(t)} = \left[\exp\left(-\mu_\lambda t\right)\right]\left[\exp\left(\sigma_\lambda^2 t^2/2\right)\right]. \tag{104}$$

The short term (small t) behavior of Eq. (104) is the same as if all units had the same reliability

$$R(t) = e^{-\mu_\lambda t} \tag{105}$$

as the typical or "average" unit ($\lambda = \mu_\lambda$). However as time passes the hazard rate centroid of the population decreases as failures tend to favor removal of less reliable units. Equations (105) and (104) are plotted in Fig. 25 to permit this effect to be displayed graphically for the case $\sigma_\lambda = 0.25\mu_\lambda$ (25% dispersion of the hazard rates). The two curves differ very little.

Before continuing we should notice that Eq. (104) exhibits a minimum at $t = \mu_\lambda/\sigma_\lambda^2$ and diverges at $t = \infty$. This anomalous behavior is due to the finite area under the left tail of Eq. (102) representing a small probability of hazard rates near zero and even negative. This is of little practical concern and use of Eq. (104) is proper for $\mu_\lambda t \ll (\mu_\lambda/\sigma_\lambda)^2$, the situation shown in Fig. 25.

To decide whether nominally alike components are nearly identical from a reliability point of view or exhibit significant dispersion, it will be necessary to distinguish profiles like the smooth curves shown in Fig. 25. The step

56

function also displayed in the figure represents a Monte Carlo simulated measurement involving 10 units. Ten hazard rates $\lambda_i$ distributed per Eq. (102) were chosen. Then since the cumulative single component unreliabilities $U_i = 1 - e^{-\lambda_i t}$ are uniformly distributed (see Ref. 19 pages 62 and 63), the simulated failure times are given by

$$t_i \;=\; -\frac{1}{\lambda_i} \ln\left(1 - U_i\right), \tag{106}$$

where the $U_i$ or $1-U_i$ are random numbers on the interval 0 to 1. What I call a poor man's Monte Carlo simulation was employed for constructing Fig. 25. Thus the random numbers were generated essentially by throwing darts at a telephone book rather than via a fancy computer algorithm. The associations of particular $\lambda$'s and $U$'s were also established by chance. The step function of Fig. 25 is only representative and not unique. Repeating the simulation will yield a different detailed outcome. The same is of course true of actual experiments yielding time-to-failure data. We can see that a very refined experiment indeed is required to select one of the curves of Fig. 25 as preferred over the other. One cannot conclude that reliability dispersion effects are fundamentally indistinguishable. But as a practical matter for items expected to be exponentially reliable, significant variations in the reliability parameters of "similar" items are not likely to be observed via the customary cataloging of times to failure under similar test conditions. This is not to be construed particularly as an argument favoring the probabilistic design approach to reliability discussed above or the physics-of-aging posture advocated by Thomas[38]. We are simply trying to characterize and develop insights relating to the macroscopic approach to reliability evaluation. The stochastic aspects of the problem preclude finding answers to questions that are too detailed. On the other hand probing exactly these informational limits is the price of progress.

Earlier we argued that reliability statements about an individual item could be made only by studying a population of similar units. Now it appears that the required similarity is very difficult to demonstrate. It seems that we have come full circle in the sense that observations of individual units may serve only to collectively characterize a population. The situation is probably not as gloomy as it begins to sound. Very likely it is easier to build similar components through meticulous control of manufacturing processes than it is to demonstrate that this has been done. In any event there are several lessons to be learned from this. The statistical nature of the problem should temper the kinds of questions one asks. Reliability experiments should be carefully thought out with respect to the relevancy and adequacy of the information to be developed. In thinking macroscopically about reliability problems it is often helpful to relax the tendency to look for rigid associations of cause and effect. Statistical problems are what they are largely because of their indeterministic features. Fortunately most reliability analysis methods do not depend on a priori product similarity. Assurance of similarity is needed only if one wishes to make sharp statements about the expected performance of an individual item based on population studies. In the case of sharply clustered wearout times to failure in parallel tests, the data record itself provides this information. For the random hazard situation the analogous connection is very weak (Fig. 25) and one needs to insist that the units be of "identical" manufacture.

57

## 8.3    Data Requirements

To be consistent with the general scope of this recommendations section, data requirement guidelines rather than comprehensive responses to particular situations are suggested. In gathering information from which inferences relating to product reliability and life are to be drawn one often tests nominally identical items under controlled conditions. This is done against a sharply defined standard of acceptable performance. Failure may be taken to be any departure of an operational or physical parameter from the established norms. Monitoring the properties of interest and comparing with the relevant failure thresholds yields a set of times at which failures occur. This kind of time-to-failure information is the preferred form from which to construct distributional analyses from the macroscopic viewpoint. In dealing with deployed sonar transducer arrays, opportunities for evaluation may be quite restricted. This naturally leads to cumulative failure information in a failures-per-interval format. The approach is quite instructive provided 100% testing is carried out at each checkpoint.

Inputs to the probabilistic design approach to reliability evaluation are quite detailed as has been mentioned above. Kececioglu documents some of these needs in Refs. 39 and 40. Basically one requires distributional information on applied stresses, component strengths, failure governing criteria, and a variety of environmental, processing, and materials characteristics. Kececioglu[39,40] has enunciated an appeal to the engineering community to improve upon the limited availability of information of this kind.

The use of Bayesian inference methods in dealing with reliability problems begins with the construction of a prior distribution. This requires previous experience with the same or similar types of hardware. A more consistently articulated evaluation of fleet operations than has been carried out previously may be required, but the inventory of wet-end sonar equipment seems well suited to the application of the Bayesian approach.

All three reliability analysis methods mentioned in this section ideally can be arranged to imply a time-to-failure probability density function type of description. The form of this function is of course very suggestive in classifying phenomena leading to failure. Thus to some extent corrosion, fatigue, etc. often exhibit generally characteristic signatures. One can look for microscopic confirmation of these by studying basic physical processes, i.e. via failure analysis.

## 8.4    Product Upgrading Strategies

Probably the single most important consideration involved in improving the reliability of military hardware is to officially recognize that there is a problem. When this is done personnel are encouraged to catalog, dissect, and interpret observed failures and the basis for understanding the causes is established. Improvements can grow out of such an appreciation of the situation. Naturally the most constructive way that this sort of feedback can impact hardware configurations is early in the design phase. With respect to weapons systems at least the Department of Defense has formally adopted this posture by issuing Directive 5000.40. This document (discussed in Ref. 41) restructures

procurement procedures as they relate to achieving reliability and maintainability as well as performance objectives. The directive does two things--it recognizes the problem and calls for solutions to be developed beginning with the earliest engineering phases of a procurement program.

For its part the Navy has adopted a very progressive approach at top management levels largely in the person of W. J. Willoughby, Jr., Deputy Chief of Naval Material for reliability, maintainability, and quality assurance. Willoughby's approach is detailed in a recent interview in Ref. 42. Basically he feels the evidence now strongly supports the contention that engineering discipline and manufacturing controls are better methods for achieving reliability than is some form of proof testing. Willoughby's posture seems to be very flexible. Contractors' ingenuity and creativity are allowed to blossom rather than being rigidly restrained.

Thus far in this section we have not discussed specific upgrading techniques such as surface preparation, burn in, or process temperature control. These specifics grow out of a more fundamental commitment to success by the people involved with a given project. And in fact the techniques just named relate to two distinctly different philosophies of achieving the desired results. Burn in is an example of the flaw precipitation approach. Components are regarded as vulnerable to the inclusion of flaws--defects which deteriorate into failures with repeated application of stress. Burn in is designed to promote these incipient failures and preempt inferior units from seeing actual service. Surface preparation, process temperature control, and chemical quality control are examples of steps taken during manufacture to avoid flaws in the final product. The upgrading strategy is to make a superior product through strict process controls rather than to select accidently better units by a post-fabrication sorting method. One hundred percent testing may still be desirable, not to induce failures, but to deduce which units never worked to begin with.

Some additional observations can be made regarding efforts to use testing to actually improve product reliability directly. Recall that components that exhibit truly exponential reliability are not degraded by use until their intrinsic strength limits are reached. Thus protracted low level testing is of no consequence. Proof testing in this case should be brief involving only loading to the maximum stress levels of interest. Good units are not damaged by this; inferior specimens are destroyed. This procedure doesn't work in wearout situations. Wearout is characterized by the accumulation of damage under extended use. Thus protracted testing or perhaps a judiciously designed accelerated test is required to demonstrate wearout reliability. However, passing such a test leaves hardware heavily aged and unfit for its intended application. Wearout testing serves to characterize similar equipment rather than qualify the particular items tested. Essentially the reverse is true for exponentially reliable components although some insights concerning expectations for similar items would also be developed in this way.

An essential recommendation that comes from all of this is that the proper role of testing is diagnostic. Combined with failure analysis it helps one identify what areas need improvement. The improvement should be accomplished by design change, material selection, altered processing, etc. not by more testing. In the case of long lived sonar equipment there may be cases where

59

improvements can be made in response to test information that seems to be rather incomplete. That is, solutions to a problem can be developed more easily than would be a full characterization of the reliability impact of the situation.

Another recommendation for upgrading hardware is to get the manufacturers constructively involved. Don't force the response of contractors to be adherence to some (perhaps obsolete) bureaucratic norm. Instead communicate objectives and let the contractors' engineering staffs determine how to meet them.

## 8.5     Incentives

In this section no specific answers or solutions are provided. We simply wish to focus attention on a continuing need if reliability benefits are to be most effectively realized. It has been stated that front-end investment in reliability produces a ten-fold payback in maintenance and repair expense avoided later. The exact figure depends on the specific situation but is nevertheless significant. For top-level procurement managers this in itself ought to be a splendid incentive. But how are middle managers and junior operatives rewarded if they save the average taxpayer a few dollars? And what is the attitude of the hardware vendor? By building better equipment does he reduce his level of repeat business? If so this is a negative incentive. If the corporate executive even suspects (correctly or not) that a better product has an adverse economic impact in his area of responsibility, he will not be expected to work for improved reliability. Thus a workable benefit situation needs to be defined at the level of every relevent profit center.

There are those idealists for whom the opportunity to do good work is its own reward (The author likes to think of himself as such.). But for the most part our political and economic systems are based on the notion that services should be inspired by and rewarded with some sort of (hopefully equitable) wage or its equivalent. In the military equipment area we cannot afford to tolerate unreliability and it debilitating side effects.[42]. But if the needed reliability is to be achieved, the economic pie must be sliced in such a way that personnel at all levels on both the consuming and producing sides recognize that the common good is in their personal best interest. As Willoughby[42] points out the bottom line is quality, ultimately the quality of the people committed to the success of the venture. The average worker is not going to be motivated simply by a chance to cast his lot on one side or another of a nebulous ideological struggle. Other ways need to be identified. In discussing this informally with Navy personnel, the author has found some reluctance to take the idea of economic incentives seriously. There seems to be precedent for this approach, however. I have been told the Canadian Air Force pays some kind of premium if equipment purchased from the United States exceeds specified reliability objectives.[43] Developing an effective incentives posture may be the single most significant way in which Navy procurement managers can impact the reliability problem.

## 8.6     Management Needs

In this section we discuss attributes that would serve well, individuals charged with upgrading the reliability of hardware being procured. The reader

should begin to realize from this report alone that the quest for reliability
is complicated by the nature of the reliability problem. As a discipline
reliability is subtle, tricky, heavily mathematical, statistically based, and
enormously important. A procurement manager does not need to be a thorough-
going reliability specialist (reliability is not his only concern) but he
should be sufficiently accomplished to obtain competent help and avoid being
bamboozled by fast-talking associates or adversaries. In the author's opinion
there is a great deal of conventional wisdom being misapplied in the name of
reliability these days. Thus the procurement manager needs to be able to cut
to the heart of relevant issues and to be capable of forming independent judg-
ments. In addition it is necessary to define realistic objectives in connection
with any particular program. There are of course various levels of reliability
management ranging from overall policy determination to incentives development
and supervision of technical implementation tasks. At the higher levels, of
course, most operational details are left to others. Nonetheless top management
people need to be conscientious and well informed. Their decisions have consid-
erable impact.

Another factor relating to the kind of talent needed in reliability man-
agement is the dynamic character of the field. Methods development and refine-
ment are continuing areas of activity. Failure analysis, probabilistic design,
accelerated testing, and Bayesian inference are all evolving areas. The need
for continuing education is apparent. Happily much is being done to meet this
need. Many fine textbooks and a great number of periodical publications treat
a wide variety of reliability topics (Access to an enormous literature is gained
by referring to the secondary sources cited by entries in the reference section
of this report.). This author is largely self-taught using such materials but
can also recommend a number of very beneficial institutes, seminars, and short
courses sponsored on a continuing basis by The University of Arizona, The
George Washington University, and the Reliability Analysis Center. The lat-
ter is a division of The Illinois Institute of Technology housed at Griffiss
Air Force Base, Rome, New York. The continuing education offerings are timely,
incisive, and in some cases directed specifically to management issues.

61

## 9.0 CONCLUSIONS

This report has been prepared with the intention of providing an integrated overview of the reliability field for technical and managerial personnel concerned with upgrading wet-end sonar equipment. An attempt was made to present the material in sufficient detail to permit the reader to digest and interpret other work and put into perspective, problems in his own particular area of interest. I cannot unilaterally say that this effort has been successful. Such a determination awaits the collective judgment of users of this document. For the author at least this study has served as a probe of the scope of the reliability problem generally and the very large amount of work surrounding it. Only a bit of scratching at the surface of this body of information has been accomplished in these pages. Nevertheless the author feels that such a step is necessary to stimulate the kind of dialogue that will lead to creative solutions to reliability problems in the specialized sonar field.

Distinctions relating to testing such as whether tests are time terminated, failure terminated, censored, or accelerated have not been sharply drawn.

Many specific reliability situations of interest have necessarily been completely ignored in the report. However a variety of subject areas and source materials are identified for those who want to pursue particular aspects in greater detail. It is the author's impression that there is considerable operational level misunderstanding about what can and can't and should and shouldn't be done in meaningfully addressing reliability problems. The reader is cautioned to guard against pitfalls of this nature and hopefully provided with some of the tools needed to make critical judgments.

Quite fortunately a strong commitment to superior hardware reliability has been made by the Navy at the top levels of management. This has taken shape for weapons systems in the "New Look" philosophy emphasizing the incorporation of reliability and maintainability efforts in the design phase of hardware procurement. Encouraging preliminary results are becoming available for some of the earliest programs handled in this way. Both the Navy and the contractors involved are pleased with what seems to be significant progress and the way they have worked together to achieve it.

I am not sure whether the attention given to sonar transducer reliability in recent years is part of the official "New Look" or not (possibly a case of my not seeing the forest for the trees). If it is, then a new look at the "New Look" is recommended. It seems to me that the intended and proper thrust of the "New Look" philosophy is progressive, flexible, and unconcerned with the perpetuation of any conventional wisdom that has become counterproductive. In this light perhaps the common efforts to construct exponential handbook reliability models for wearout problems should be discarded as anachronistic. It is not clear that contractors do not view these exercises simply as busy work--part of the red tape associated with doing business with the Government. The special scale, longevity, and accessibility situation for wet-end sonar equipment suggests further that we concern ourselves more with actual improvements rather than emphasizing the evaluation question per se.

Not as a conclusion but simply as a concluding remark the author would like to invite and strongly encourage critical feedback from the reader concerning the usefulness of this report in dealing with his particular reliability concerns. It is often through interaction and interdisciplinary cross fertilization that collective problems are most effectively addressed.

## 10.0 ACKNOWLEDGMENTS

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Table I.  Important Reliability Functions and Relationships

Function      Name/Description

R(t)       Reliability -- Probability of system success

U(t)       Unreliability -- Cumulative failure distribution function

f(t)       Time-to-failure probability density function (p.d.f.)

$\lambda(t)$       Hazard rate -- Instantaneous failure rate

MTBF       Mean time between failures -- Expected life

R(t,T)     Conditional reliability for mission of duration T beginning
           at time t

Functional Relationships                                    Equation No.

$$R(t) = \int_{t}^{\infty} f(t)dt = \exp\left(-\int_{o}^{t} \lambda(t)dt\right) \tag{1a,1b}$$

$$U(t) = \int_{-\infty}^{t} f(t)dt \tag{2}$$

$$f(t) = \frac{dU(t)}{dt} = \lambda(t)R(t) = -\frac{dR(t)}{dt} \tag{3a,3b,3c}$$

$$\lambda(t) = -\frac{1}{R(t)}\frac{dR(t)}{dt} = -\frac{d}{dt}\ln R(t) \tag{4a,4b}$$

$$MTBF = \int_{o}^{\infty} t\,f(t)dt = \int_{o}^{\infty} R(t)dt \tag{5a,5b}$$

$$R(t) + U(t) = 1 \tag{6}$$

$$R(t,T) = \frac{R(t+T)}{R(t)} \tag{7}$$

68

Table II.  Environment and Quality $\pi$-Factors for General-Purpose Diodes

| ENVIRONMENT | $\pi_E$ |
|---|---|
| Ground, Benign ($G_B$) | 1 |
| Space, Flight ($S_F$) | 1 |
| Ground, Fixed ($G_F$) | 5 |
| Ground, Mobile ($G_M$) | 10 |
| Naval, Sheltered ($N_S$) | 12 |
| Naval, Unsheltered ($N_U$) | 20 |
| Airborne, Inhabited, Transport ($A_{IT}$) | 25 |
| Airborne, Inhabited, Fighter ($A_{IF}$) | 25 |
| Airborne, Uninhabited, Transport ($A_{UT}$) | 25 |
| Airborne, Uninhabited, Fighter ($A_{UF}$) | 40 |
| Missile, Launch ($M_L$) | 40 |

| QUALITY LEVEL | $\pi_Q$ |
|---|---|
| JANTXV | 0.15 |
| JANTX | 0.3 |
| JAN | 1.5 |
| Lower | 7.5 |
| Plastic | 15.0 |

Table III. Fifteen–Step Mechanical Reliability Prediction and Design–
for–Reliability Methodology (taken from Reference 8).

1. Define the Design Problem and Determine the Mission Profile

2. Determine the Design Variables and Parameters Involved

3. Conduct a Failure Modes, Effects, and Criticality Analysis

4. Determine the Dependence or Independence of the Component's
   Failure Modes

5. Determine the Failure Governing Criterion Involved in
   Each Failure Mode

6. Formulate the Failure Governing Stress Function

7. Determine the Distribution of Each Design Stress Variable
   and Factor for Each Failure Mode

8. Determine the Failure Governing Stress Distribution for
   Each Failure Mode

9. Formulate the Failure Governing Strength Function

10. Determine the Distribution of Each Design Strength Variable
    and Factor for Each Failure Mode

11. Determine the Failure Governing Strength Distribution for
    Each Failure Mode

12. Determine the Component's Reliability for Each Failure Mode

13. Determine the Component's Reliability for All Failure Modes

14. Determine the Overall Component Reliability Considering All
    Failure Modes Involved

15. Determine the Confidence Limit on the Calculated Component
    Reliability

Table IV. Corrosion Model Computational Parameters and Functions

| LABEL | DEFINITION | NAME |
|---|---|---|
| a | $\left(\sigma_{r_o}/\mu_{r_o}\right)^2$ | squared coefficient of variation--radius |
| b | $\left(\sigma_c/\mu_c\right)^2$ | squared coefficient of variation--corrosion rate |
| z | $\left(\mu_c/\mu_{r_o}\right)t$ | standardized process rate/time index |
| $\Delta z$ | $\left(\mu_c/\mu_{r_o}\right)\Delta t$ | rate/time increment |
| S' | . . . . . . . . | fractional strength endpoint |
| A | $1 - z$ | . . . . . . . . . . . . . . . . . . . . . . . . . . . |
| B | $bz$ | . . . . . . . . . . . . . . . . . . . . . . . . . . . |
| C | $a + bz^2$ | . . . . . . . . . . . . . . . . . . . . . . . . . . . |
| $\mu_{S'}$ | $A^2 + C$ | (1+a) times mean fractional strength |
| $\sigma_{S'}$ | $\sqrt{2}\left(\mu_{S'}^2 - A^4\right)^{\frac{1}{2}}$ | (1+a) times fractional strength standard deviation |
| $-\dfrac{d\mu_{S'}}{dz}$ | $2\left(A - B\right)$ | . . . . . . . . . . . . . . . . . . . . . . . . . . . |
| $\dfrac{d\sigma_{S'}}{dz}$ | $\left(4/\sigma_{S'}\right)\left(\mu_{S'}B - AC\right)$ | . . . . . . . . . . . . . . . . . . . . . . . . . . . |
| $\psi$ | $\left((1+a)S' - \mu_{S'}\right)/\sigma_{S'}$ | . . . . . . . . . . . . . . . . . . . . . . . . . . . |
| f(z) | $\left[\dfrac{1}{\sqrt{2\pi}\,\sigma_{S'}}\,e^{-\psi^2/2}\right]\left[-\dfrac{d\mu_{S'}}{dz} - \psi\left(\dfrac{d\sigma_{S'}}{dz}\right)\right]$ | Standardized rate/time to endpoint density function |

input data (brace spanning rows a through S')

Table V. Ordered Time-to-Failure Data and Median Ranks
(Sample Size - 20 Units)

| FAILURE NUMBER $j$ | TIME TO FAILURE $t_j$ (hours) | MEDIAN RANK $(MR)_j$ (percent) |
|---|---|---|
| 1 | 1,943 | 3.41 |
| 2 | 3,376 | 8.25 |
| 3 | 4,180 | 13.15 |
| 4 | 4,311 | 18.06 |
| 5 | 5,124 | 22.97 |
| 6 | 5,976 | 27.88 |
| 7 | 6,416 | 32.80 |
| 8 | 7,250 | 37.71 |
| 9 | 7,761 | 42.63 |
| 10 | 8,245 | 47.54 |
| 11 | 8,528 | 52.46 |
| 12 | 9,226 | 57.37 |
| 13 | 10,447 | 62.29 |
| 14 | 10,508 | 67.20 |
| 15 | 11,028 | 72.12 |
| 16 | 11,462 | 77.03 |
| 17 | 12,803 | 81.94 |
| 18 | 12,998 | 86.85 |
| 19 | 13,026 | 91.75 |
| 20 | 16,042 | 96.59 |

Table VI. Application of the $\chi^2$ Goodness-of-Fit Test to the Data of Table V.

| CLASS NUMBER | LOWER CLASS BOUNDARY (time-hours) | UPPER CLASS BOUNDARY (time-hours) | OBSERVED FREQUENCY (Number of Failures $O_i$) | EXPECTED FREQUENCY $E_i$ | $\chi^2$ STATISTIC $\dfrac{(O_i - E_i)^2}{E_i}$ |
|---|---|---|---|---|---|
| 1 | 0 | 5,500 | 5 | 4.47 | 0.063 |
| 2 | 5,500 | 8,400 | 5 | 5.29 | 0.016 |
| 3 | 8,400 | 11,300 | 5 | 4.92 | 0.001 |
| 4 | 11,300 | $\infty$ | 5 | 5.32 | 0.019 |

$\chi^2 = 0.099$

$\nu = 4 - 2 - 1 = 1$ degree of freedom

$\chi^2_{(1-\alpha),\nu} = \chi^2_{0.95,1} = 3.84$ (from published tables)

$\alpha = 0.05$

$\chi^2 = 0.099 < 3.84 \rightarrow$ Hypothesis not rejected

Table VII. Application of the Kolmogoroff-Smirnov Goodness-of-Fit Test to the Data of Table V.

| TIME TO FAILURE (hours) | RANK FRACTION OF OBSERVED DATA $S_n(x_r) = \frac{r}{n}$ | THEORETICAL CUMULATIVE DISTRIBUTION $F(x_r) = 1 - e^{-(x_r/\eta)^\beta}$ $\beta=2.3 \ \eta=10^4 \text{ hrs}$ | ABSOLUTE DIFFERENCE $\left| S_n(x_r) - F(x_r) \right|$ |
|---|---|---|---|
| 1,943  | 0.05 | 0.023 | 0.027 |
| 3,376  | 0.10 | 0.079 | 0.021 |
| 4,180  | 0.15 | 0.126 | 0.024 |
| 4,311  | 0.20 | 0.134 | 0.066 |
| 5,124  | 0.25 | 0.193 | 0.057 |
| 5,976  | 0.30 | 0.264 | 0.036 |
| 6,416  | 0.35 | 0.303 | 0.047 |
| 7,250  | 0.40 | 0.380 | 0.020 |
| 7,761  | 0.45 | 0.428 | 0.022 |
| 8,245  | 0.50 | 0.474 | 0.026 |
| 8,528  | 0.55 | 0.500 | 0.050 |
| 9,226  | 0.60 | 0.564 | 0.036 |
| 10,447 | 0.65 | 0.669 | 0.019 |
| 10,508 | 0.70 | 0.674 | 0.026 |
| 11,028 | 0.75 | 0.714 | 0.036 |
| 11,462 | 0.80 | 0.746 | 0.054 |
| 12,803 | 0.85 | 0.829 | 0.021 |
| 12,998 | 0.90 | 0.839 | 0.061 |
| 13,026 | 0.95 | 0.841 | 0.109 ← d |
| 16,042 | 1.00 | 0.948 | 0.052 |

$n = 20$

$\alpha = 0.05$

$d_\alpha(n) = 0.294$ (critical value from published tables)

$d = \max \left| S_n(x_r) - F(x_r) \right| = 0.109 < 0.294 \rightarrow$ Hypothesis not rejected
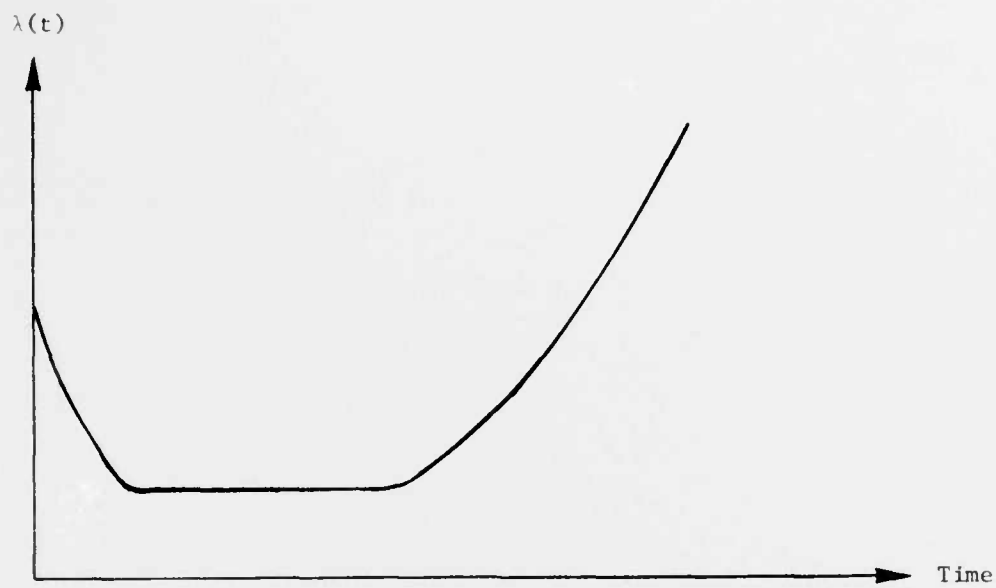
a. Electronic Components
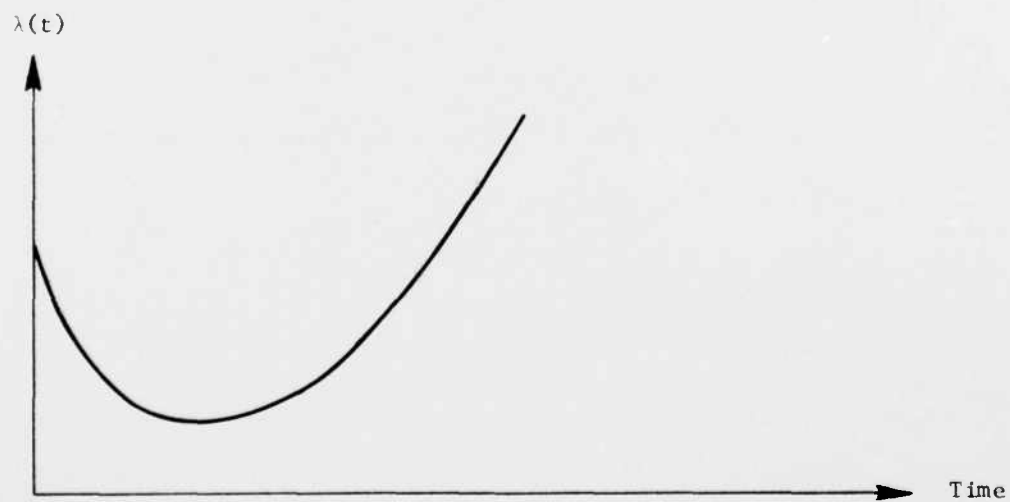


b. Mechanical Components

Figure 1. Typical Hazard Rate Functions
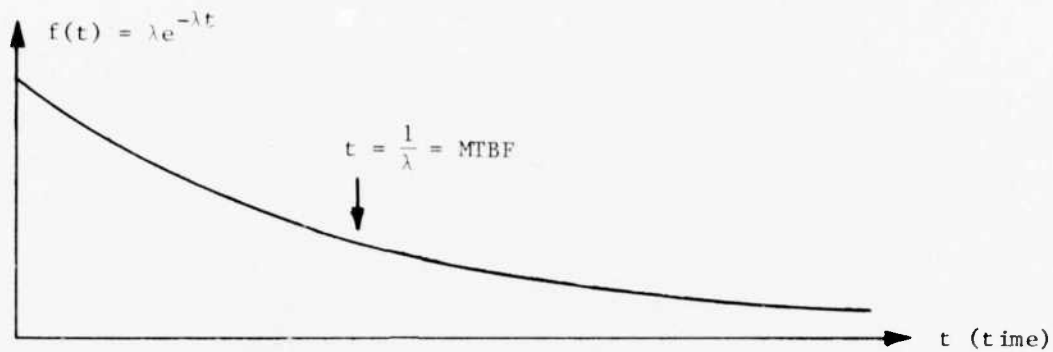
a. Exponential Time-to-Failure PDF
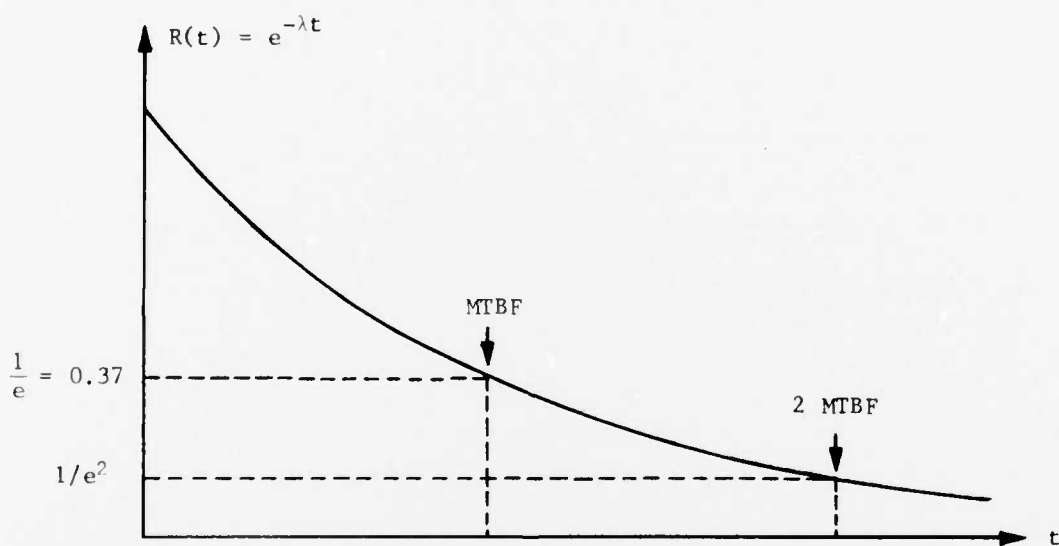
b. Exponential Reliability
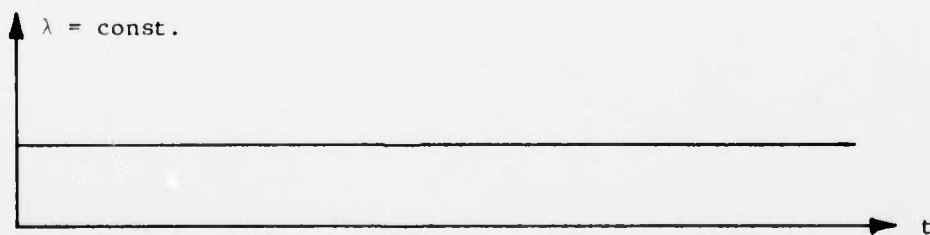
c. Constant Hazard Rate

Figure 2. Random Hazard Reliability Functions
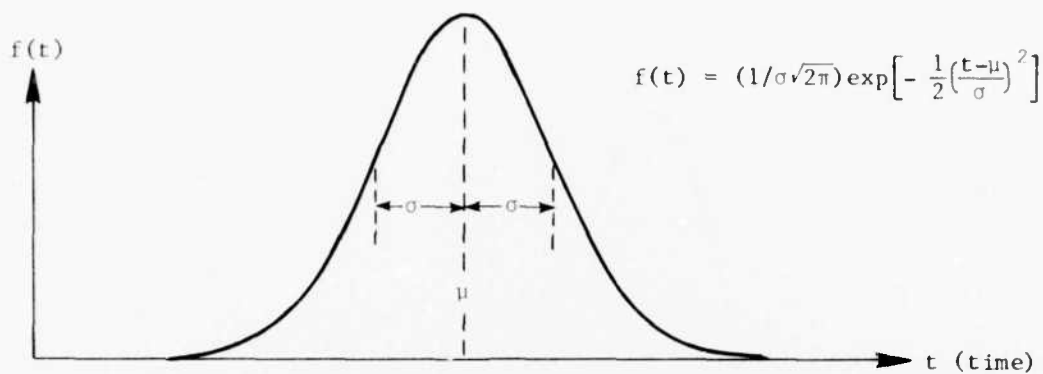
a. Gauss Normal Time-to-Failure PDF

$$f(t) = (1/\sigma\sqrt{2\pi})\exp\left[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right]$$

$$R(t) = \int_t^\infty f(t)dt$$
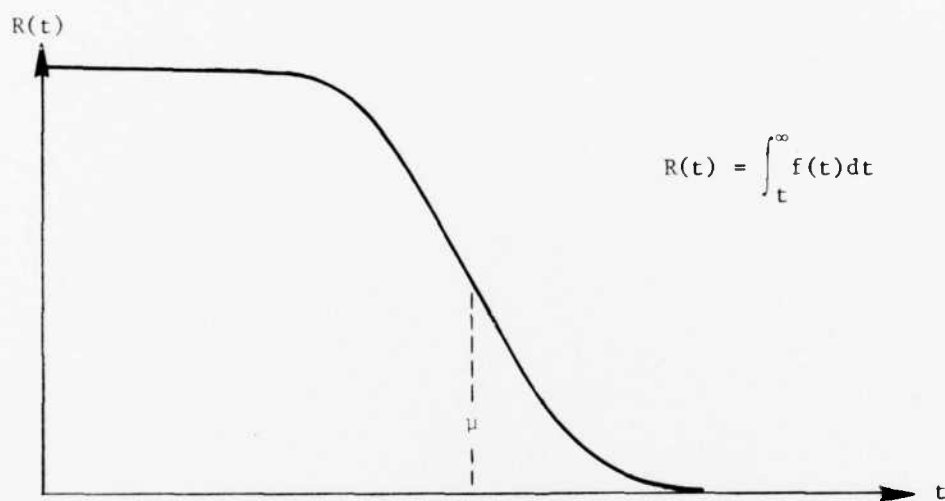
b. Wearout Reliability

$$\lambda(t) = \frac{f(t)}{R(t)}$$
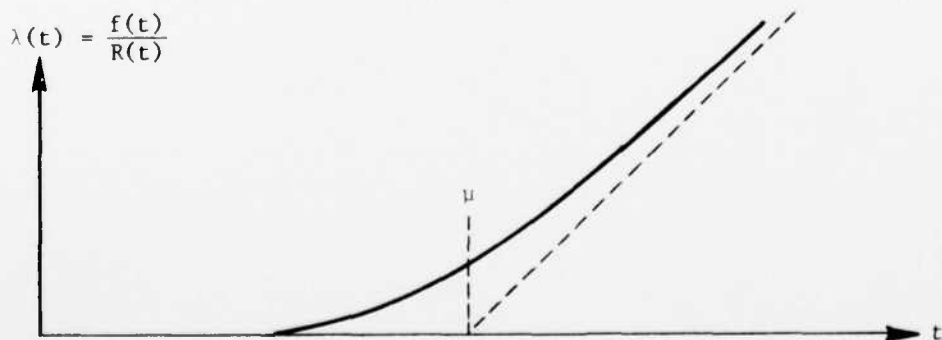
c. Increasing Hazard Rate

Figure 3. Wearout Reliability Functions (Normal PDF)

77

a. Log Normal Time-to-Failure PDF

$$f(t) = (1/t\sigma\sqrt{2\pi})\exp\left[-\frac{1}{2}\left(\frac{\ln t - \mu}{\sigma}\right)^2\right]$$

b. Reliability Function

$$R(t) = \int_t^\infty f(t)dt$$

$$\lambda(t) = \frac{f(t)}{R(t)}$$
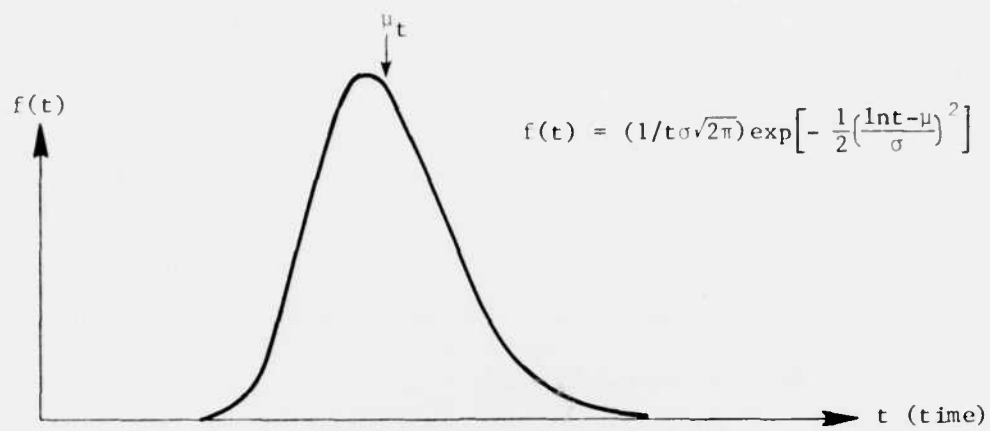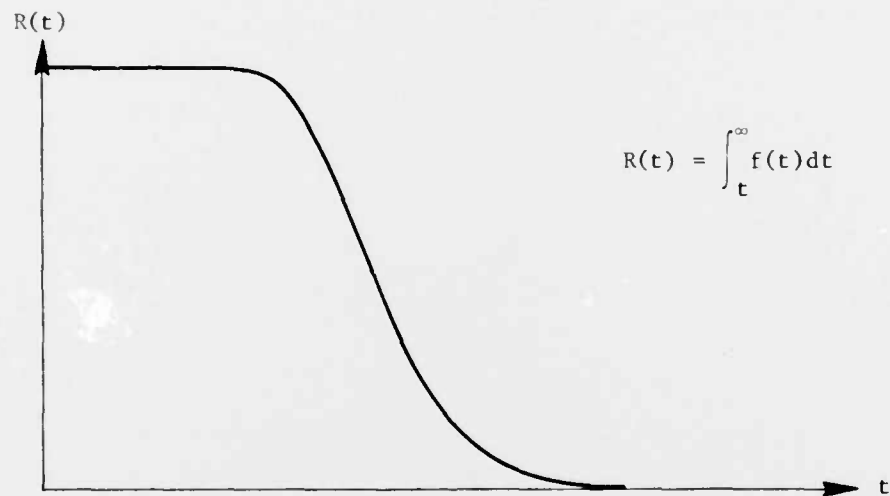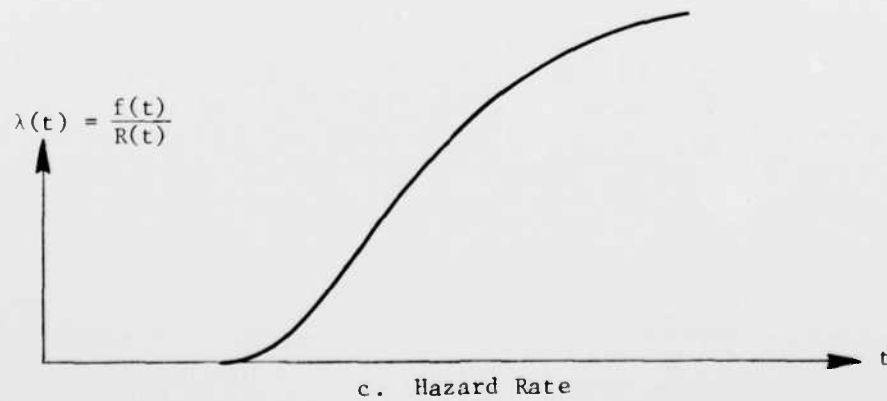
c. Hazard Rate

Figure 4. Wearout Reliability Functions (Log Normal PDF)

78

$$f(t) = 0.5t^{-0.5}e^{-t^{0.5}}$$

a.  Weibull Time-to-Failure PDF ($\beta < 1$)

$$R(t) = e^{-t^{0.5}}$$

b.  Reliability Function

$$\lambda(t) = 0.5t^{-0.5}$$

c.  Decreasing Hazard Rate
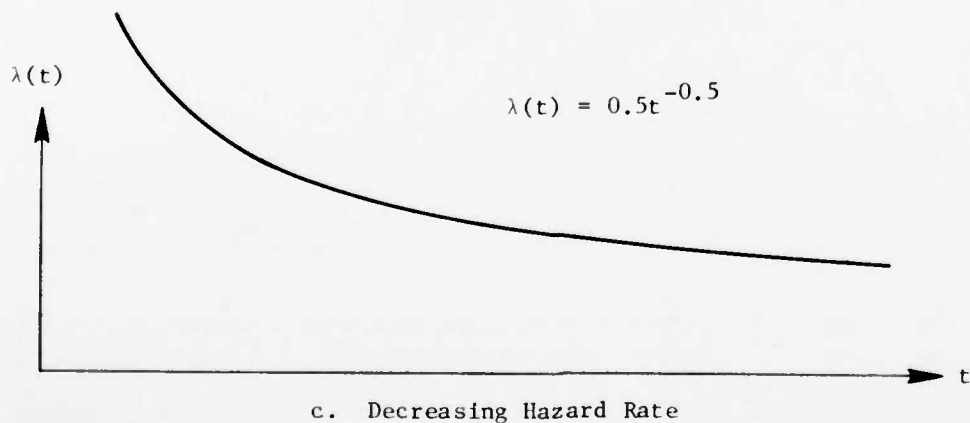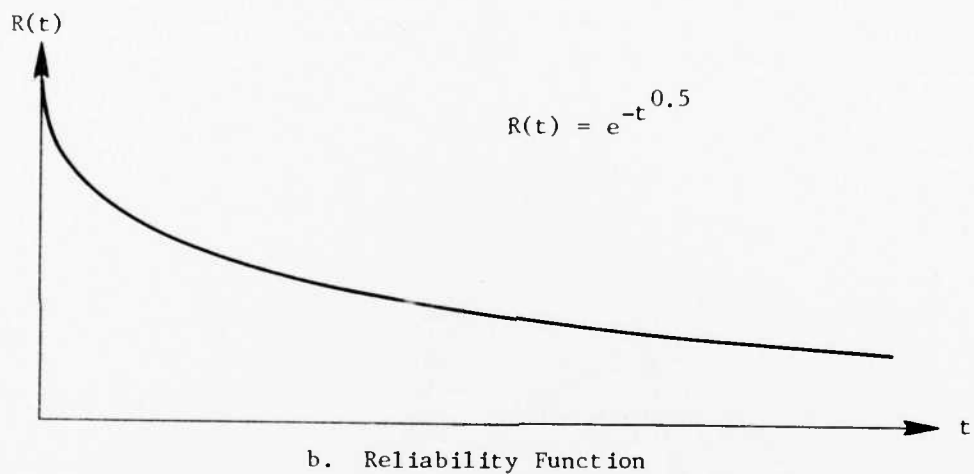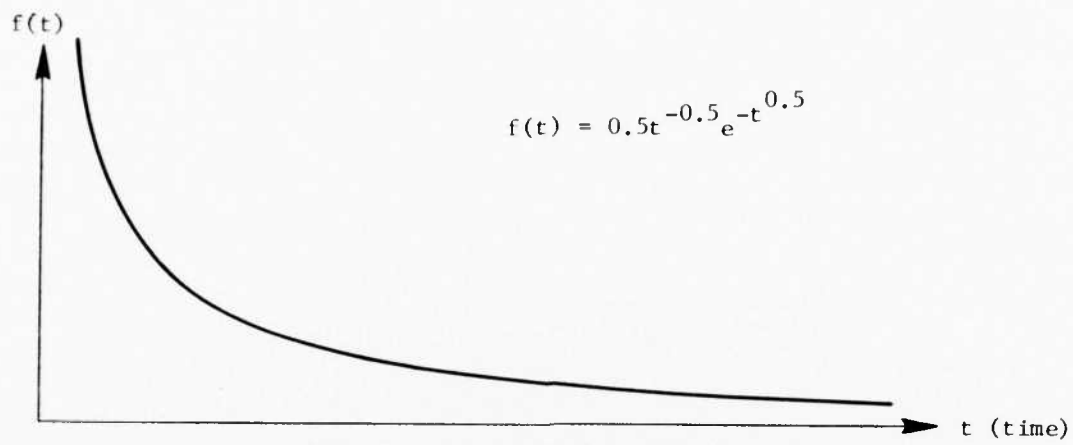
Figure 5.  Early Failure Reliability Functions

$$f(t) = \left(\beta/\eta\right)\left((t-\gamma)/\eta\right)^{(\beta-1)}\left[\exp\left(-\{(t-\gamma)/\eta\}^{\beta}\right)\right]$$
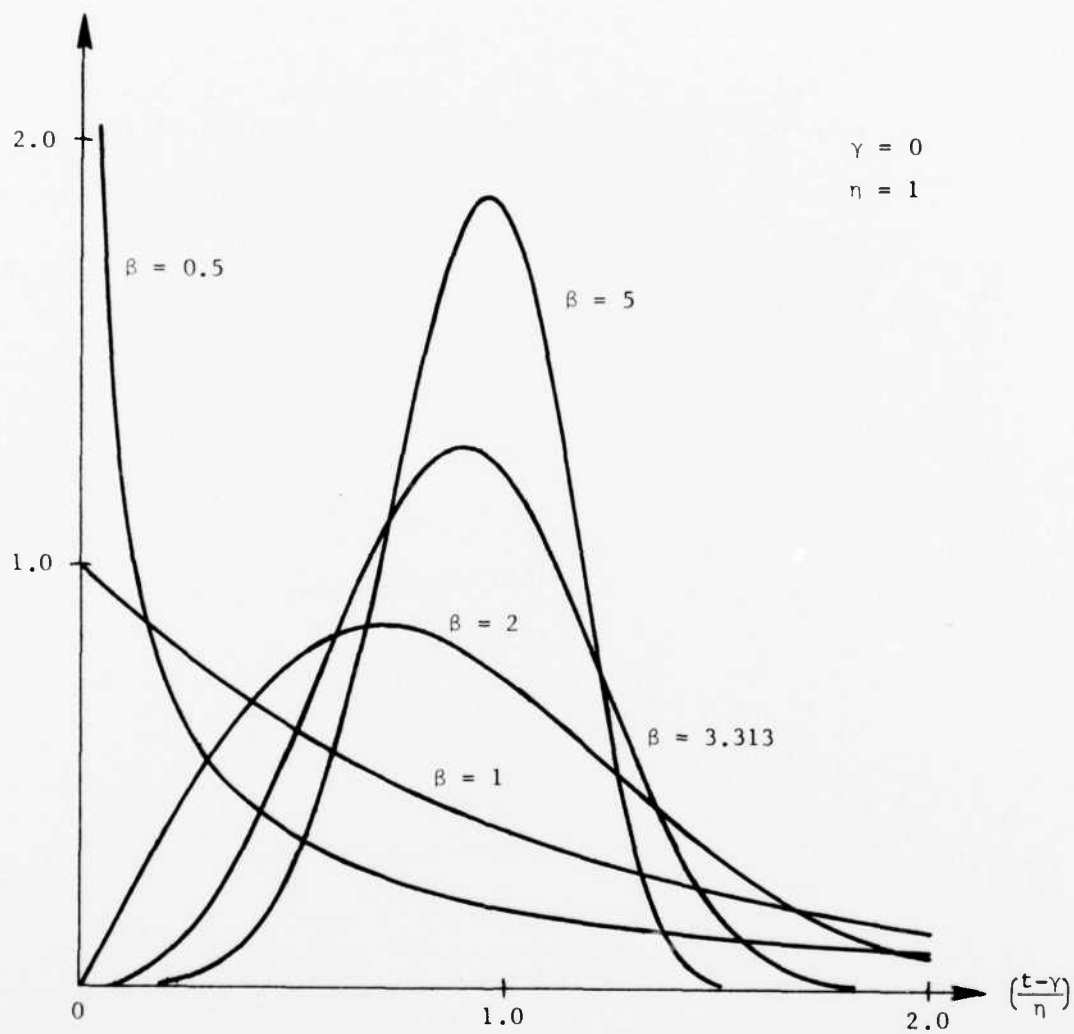
Figure 6.   Effect of the Shape Parameter β on the
Weibull Time-to-Failure PDF

80
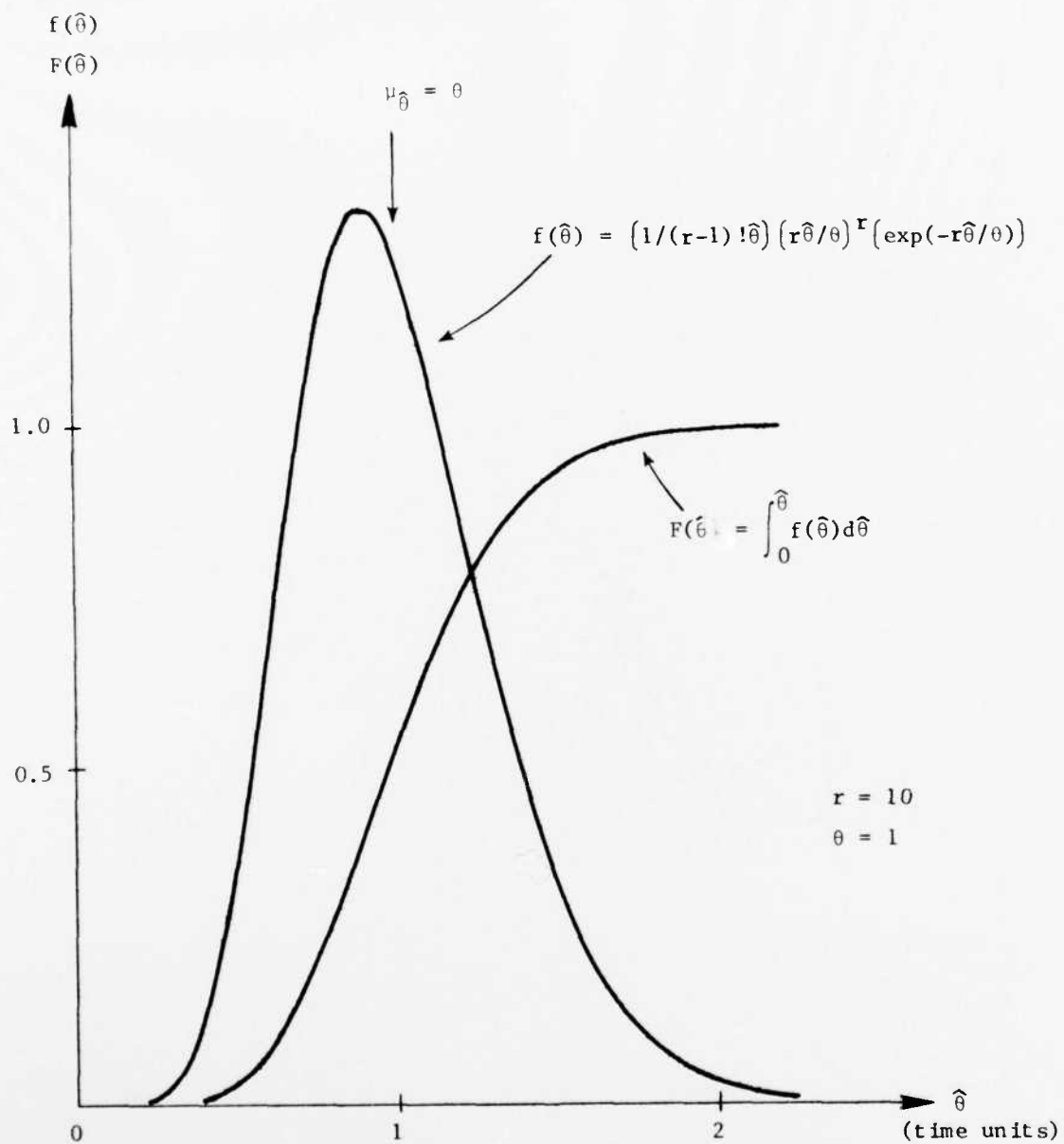
Figure 7. Probability Distribution of MTBF Estimators and Its Cumulative (Exponential Parent Population)
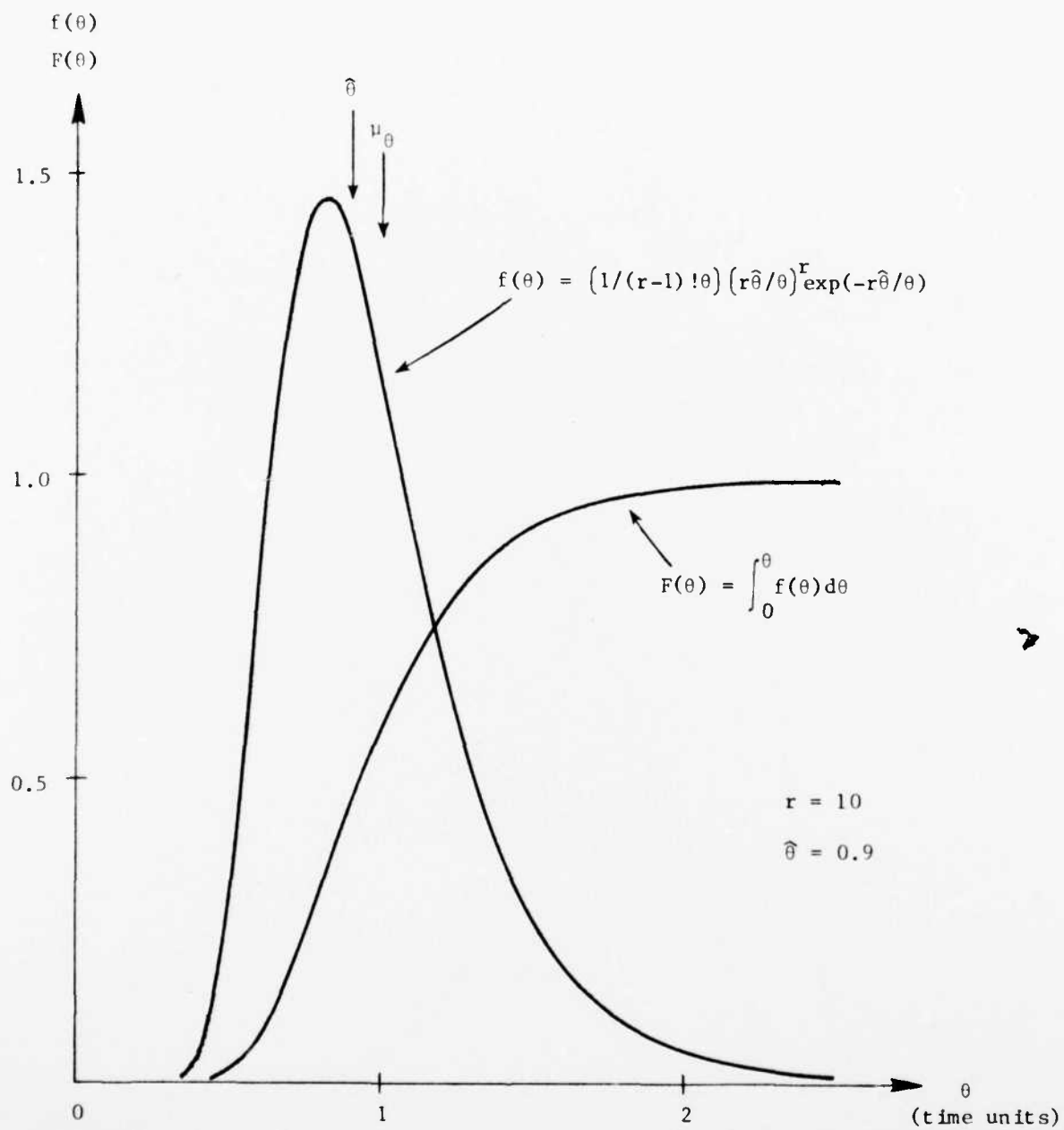
Figure 8. Probability Distribution of True MTBF and
Its Cumulative (Exponential Parent Population)

$$f(R) = \left(1/(r-1)!\right)\left(r\hat{\theta}/t\right)^r\left(-\ln R\right)^{(r-1)}\left(R^{(r\hat{\theta}/t\,-1)}\right)$$
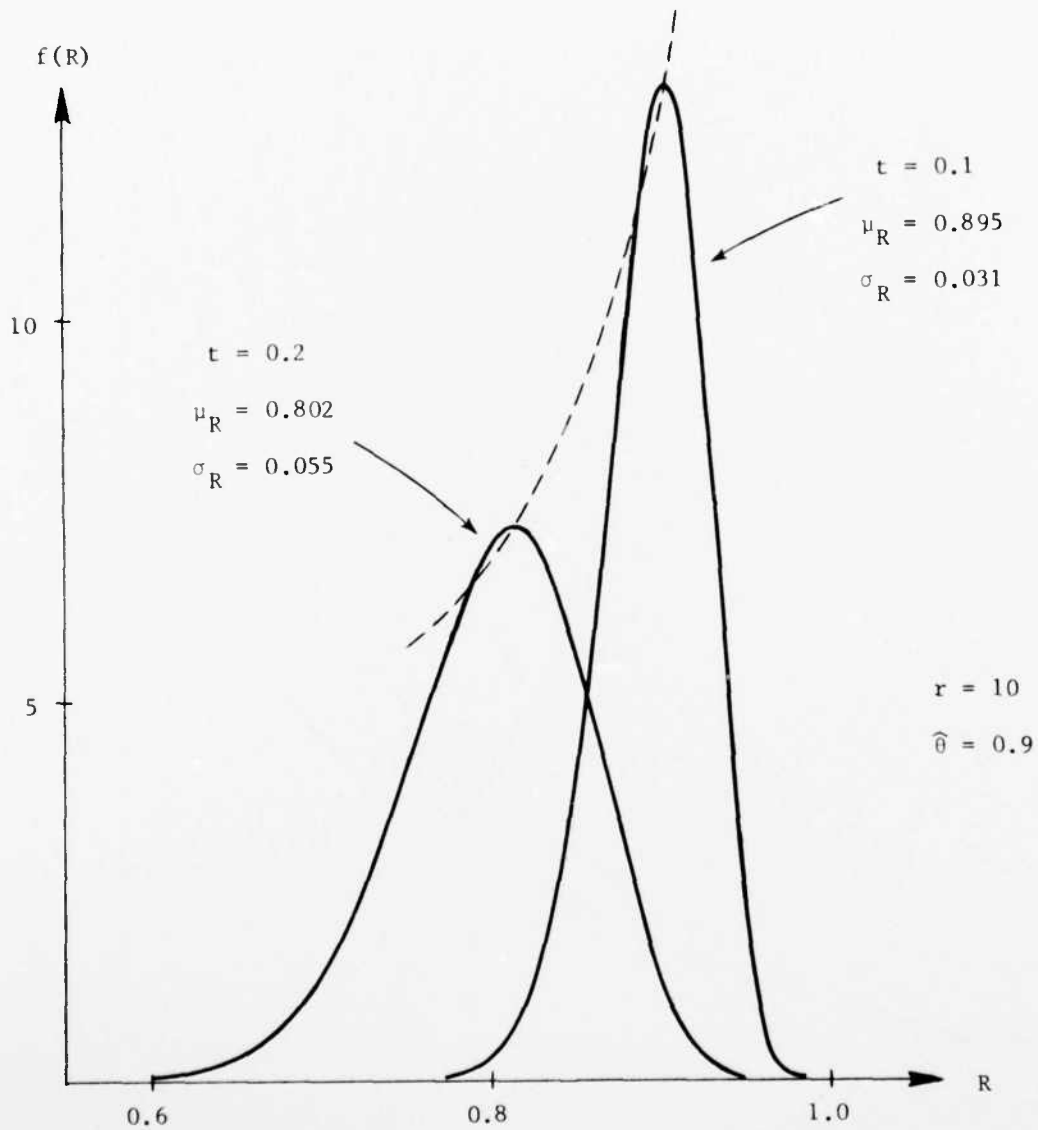


Figure 9. Probability Distribution of Reliability as a Function of Time (Exponential Parent Population)

Exponential Parent Population

10 Failures Observed

$R_L(t)$, $R_U(t)$

80%

50%

$t/\theta$
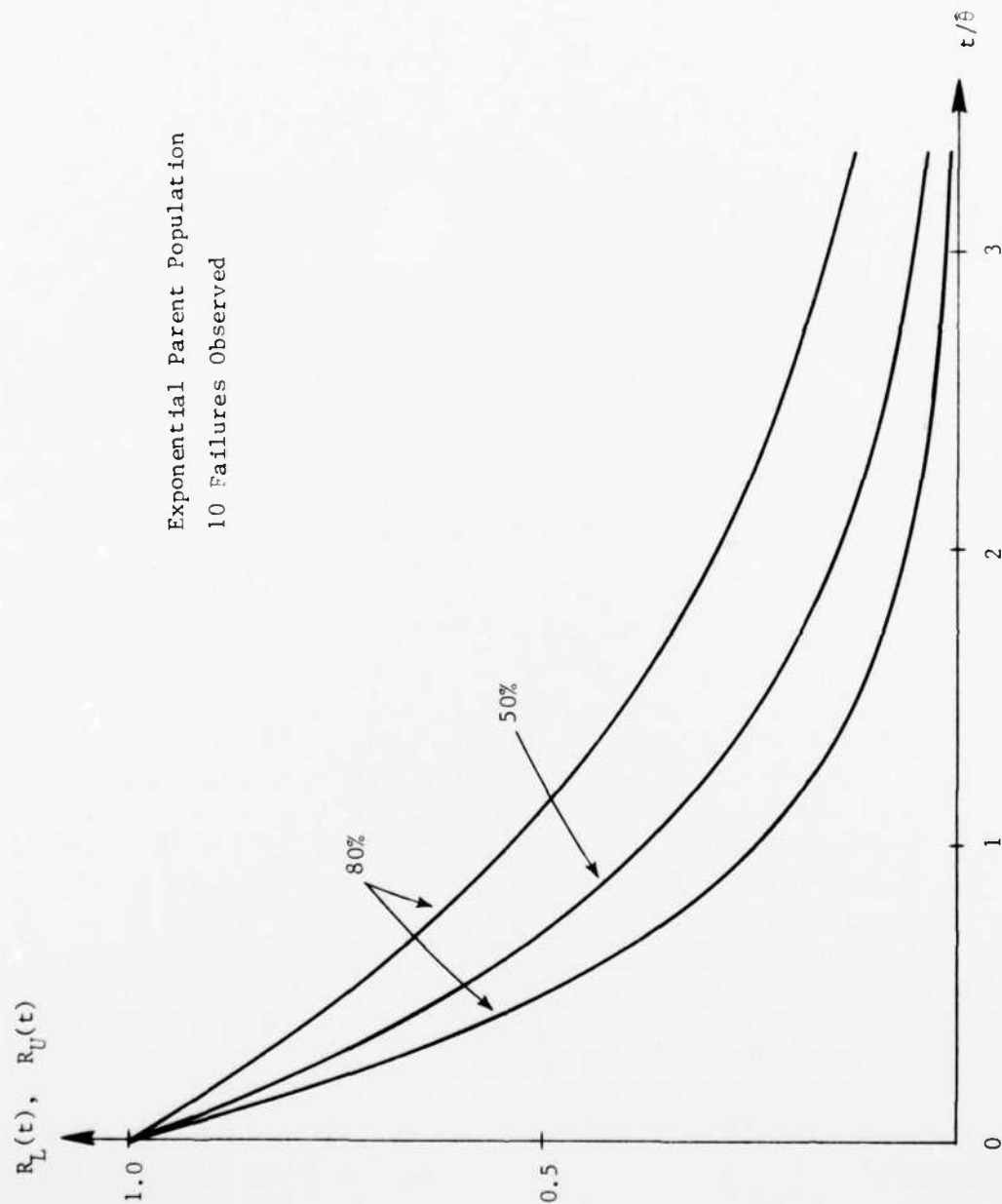
Figure 10. Confidence Limits on Reliability as a Function of Time and Confidence Level (50% and 80%)
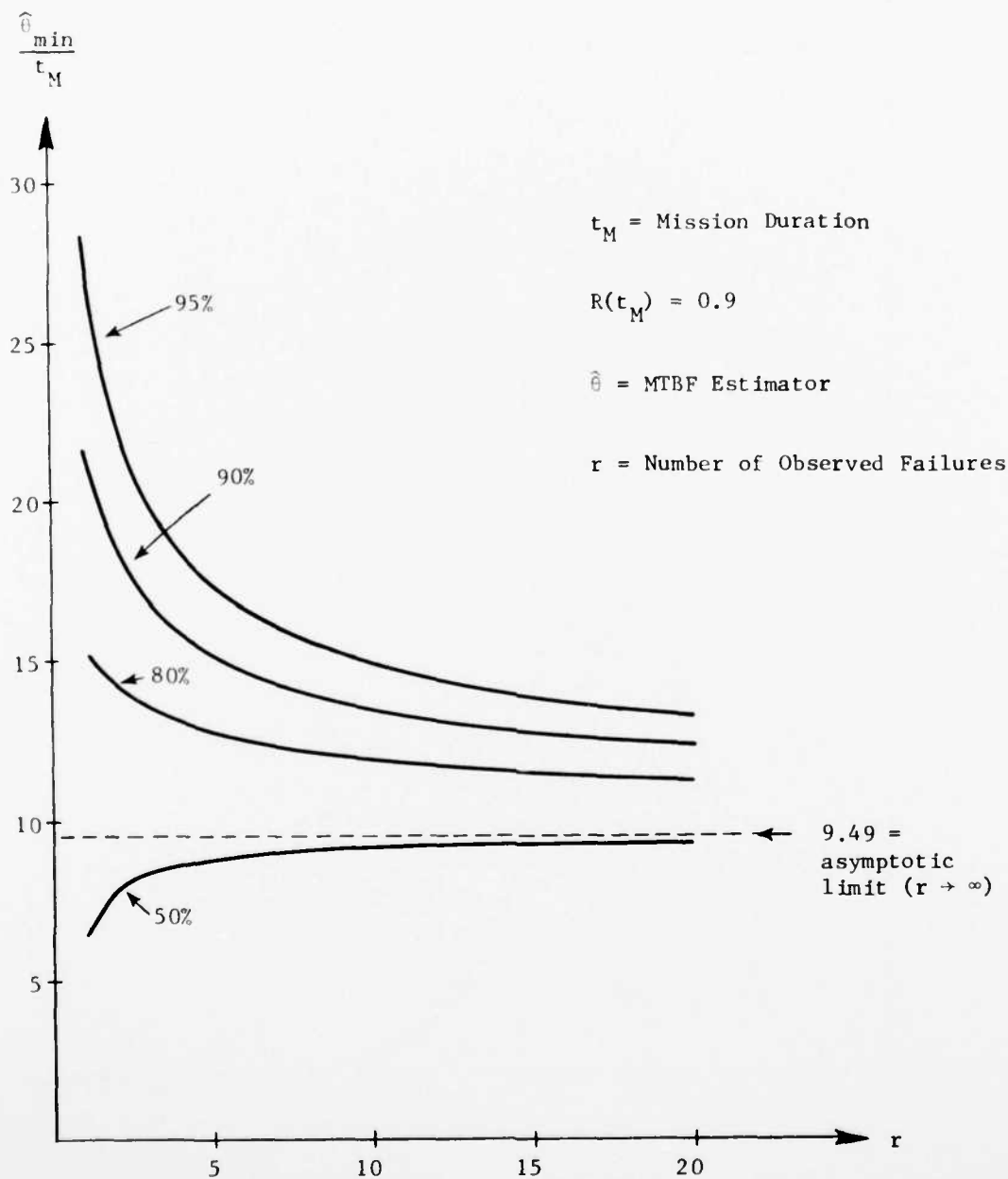
Figure 11. Minimum Acceptable Estimated (Demonstrated) MTBF
in Mission Duration Units as a Function of Number of
Observed Failures to Demonstrate 90% Exponential
Reliability at Several Confidence Levels
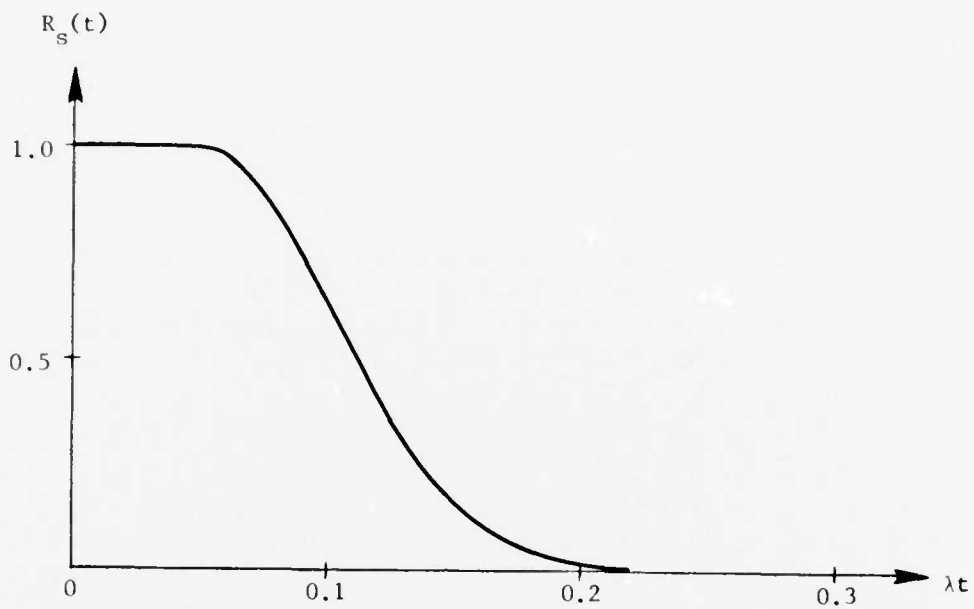
a. System Reliability
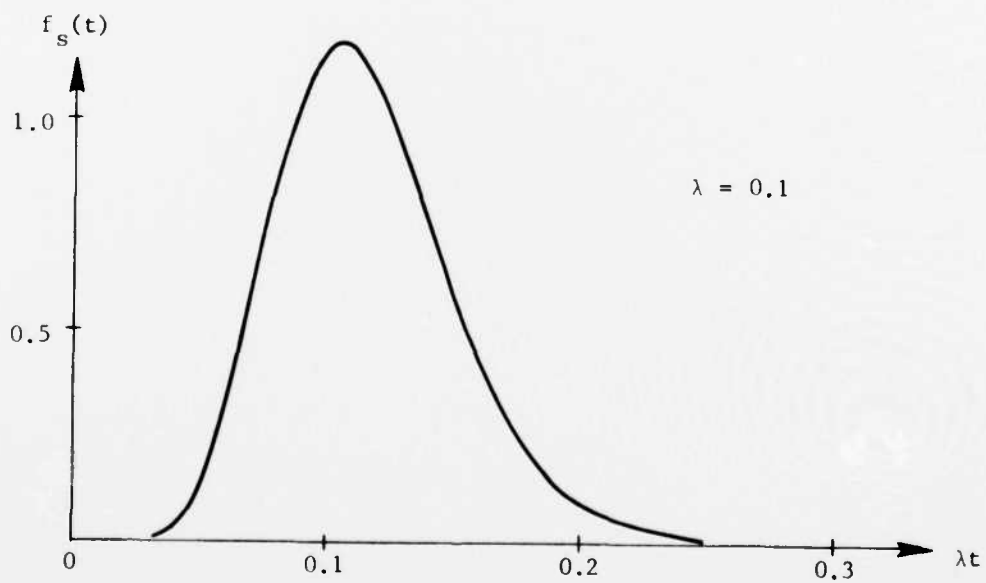
b. System Time-to-Failure PDF

Figure 12. Transducer Array System Reliability and
Time-to-Failure Density Functions

$$f_w(t) = (1/\sigma\sqrt{2\pi})\exp\left[-\tfrac{1}{2}(t-\mu)^2/\sigma^2\right]$$

$$\mu = 1, \quad \sigma = 0.2$$
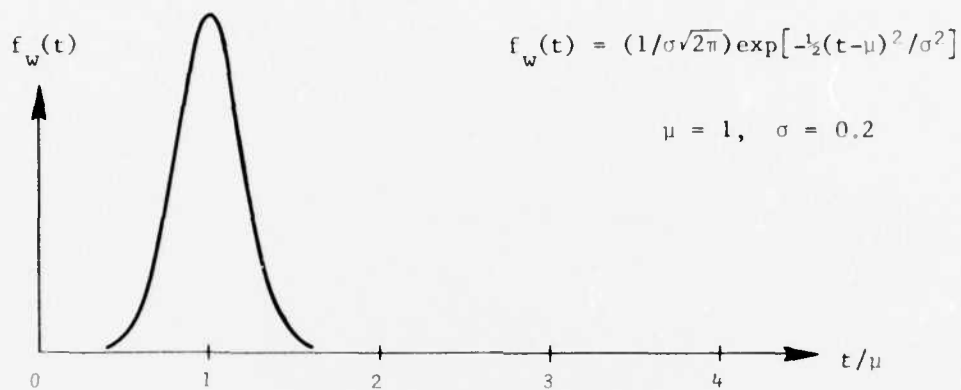
a.   Simple Wearout Time-to-Failure PDF

$$\mu \rightarrow n\mu, \quad \sigma \rightarrow \sqrt{n}\,\sigma$$
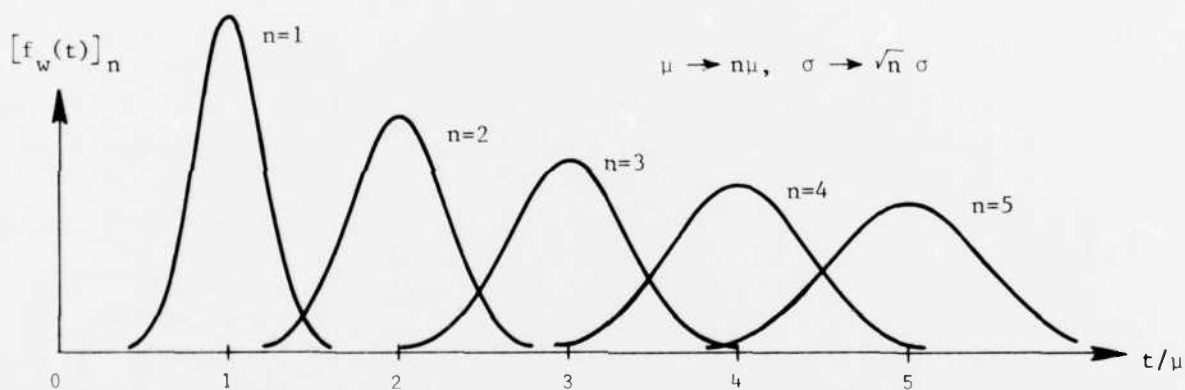
b.   Wearout PDF Under Continuous Replacement (Generations Isolated)
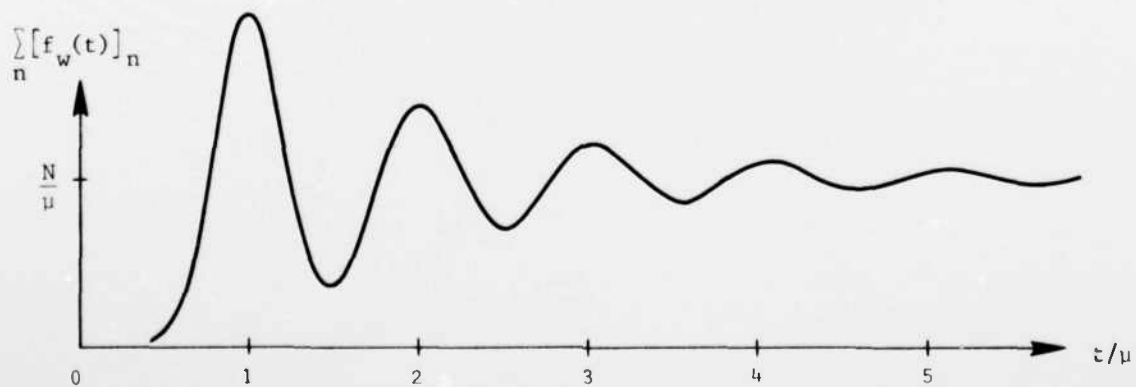
c.   Superposition of the Components of Part b

Figure 13.   Failure Frequency Under the Continuous Replacement
of Wearout Failures

s = Stress

S = Strength

$$f'(s) = f(s) \int_{-\infty}^{S} f(S)dS = \text{Stress-to-Failure PDF}$$

$$Q = \int_{-\infty}^{\infty} f'(s)ds = \text{Unreliability}$$
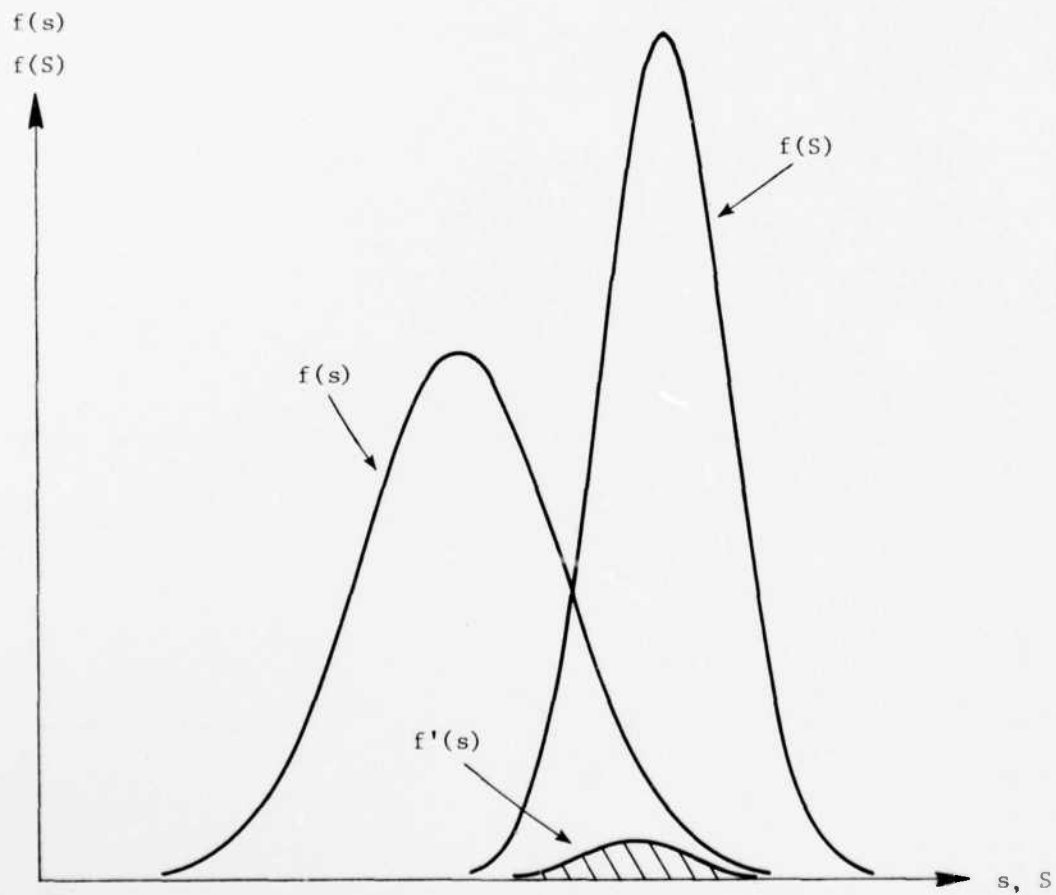


Figure 14. Illustrative Example of Overlapping Stress and Strength Distributions
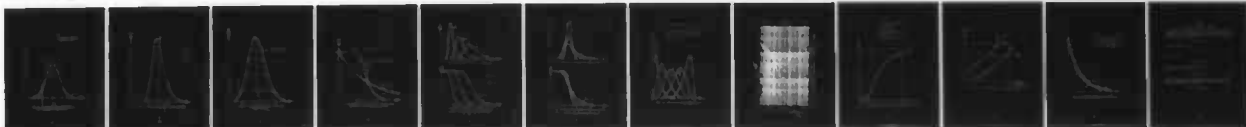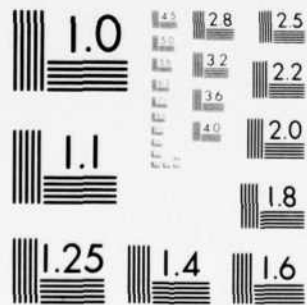
S = Strength

U(S',t) = Time-Dependent Unreliability
(Fraction Worn Out)
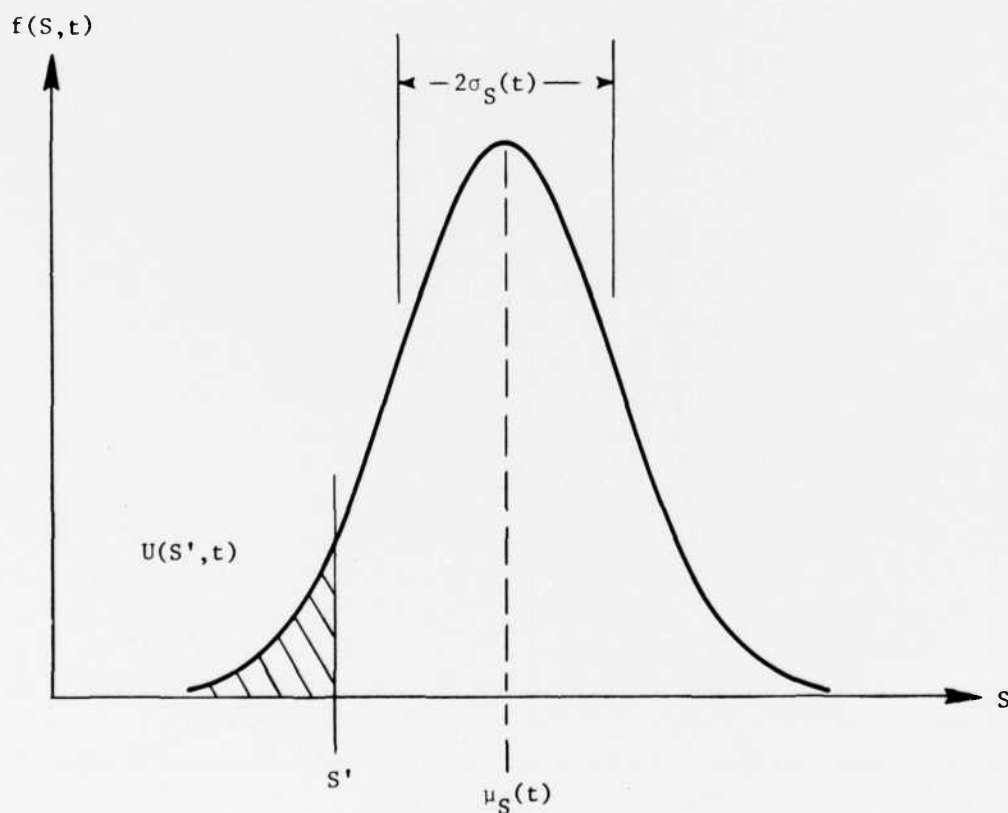


Figure 15. Time-Dependent Strength Distribution
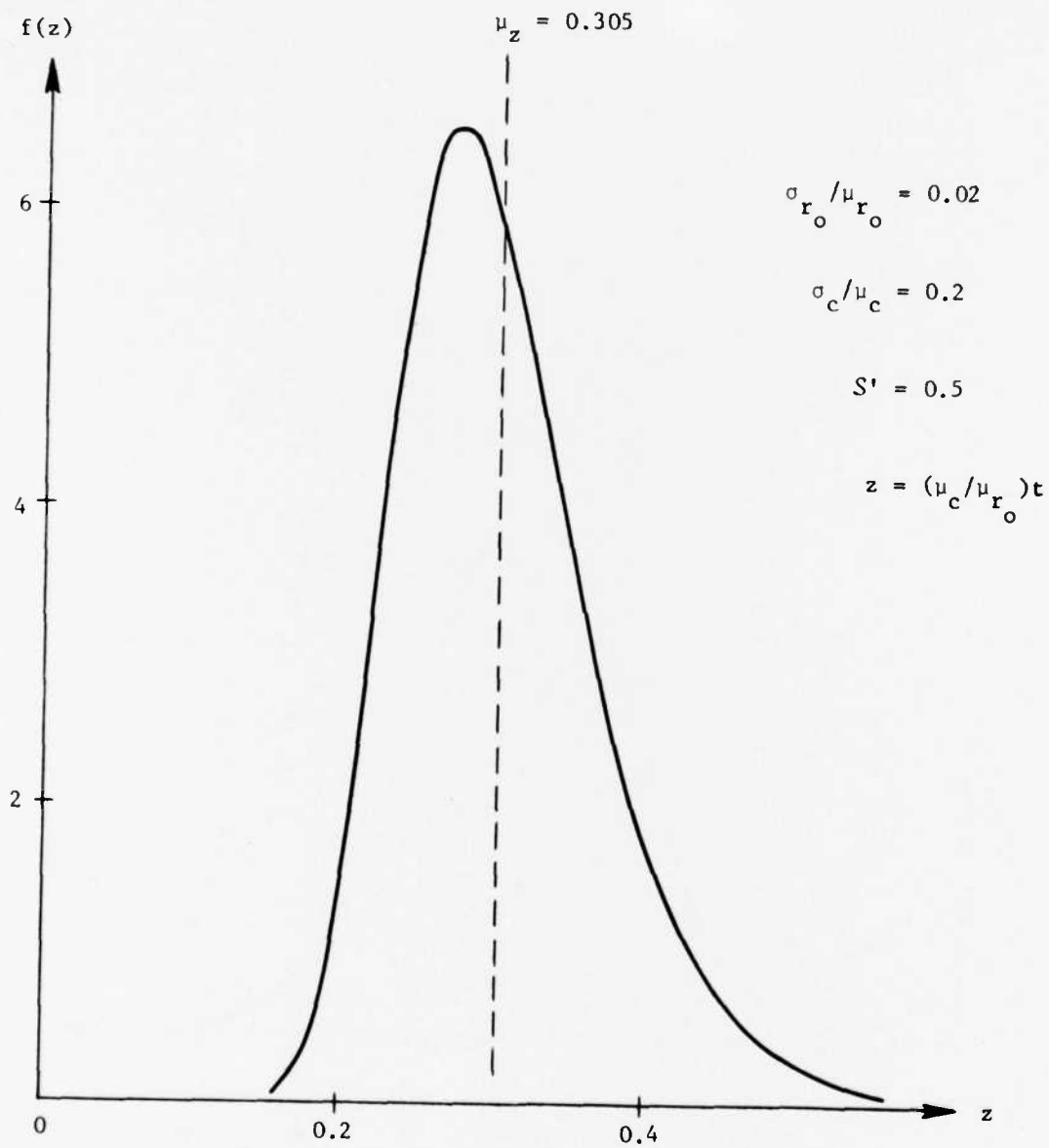f(S,t) Impinging on Service Limit S'
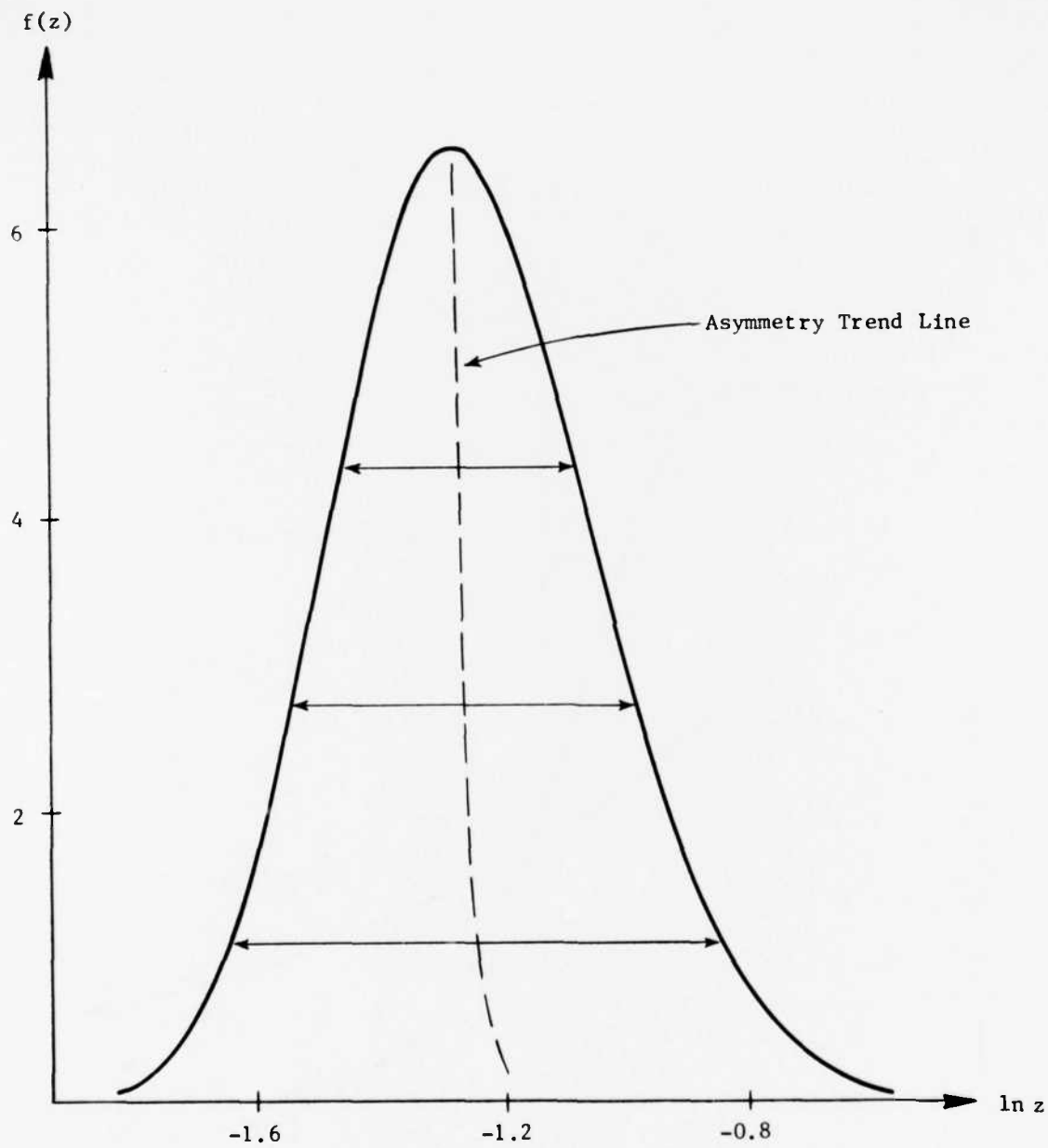
Figure 16.  Typical Corrosion Model Time-to-Failure PDF

90

Figure 17. Logarithmic Plot of Time-to-Failure PDF
(Figure 16 Data)

91

$$\sigma_{r_o} / \mu_{r_o} = 0.02$$
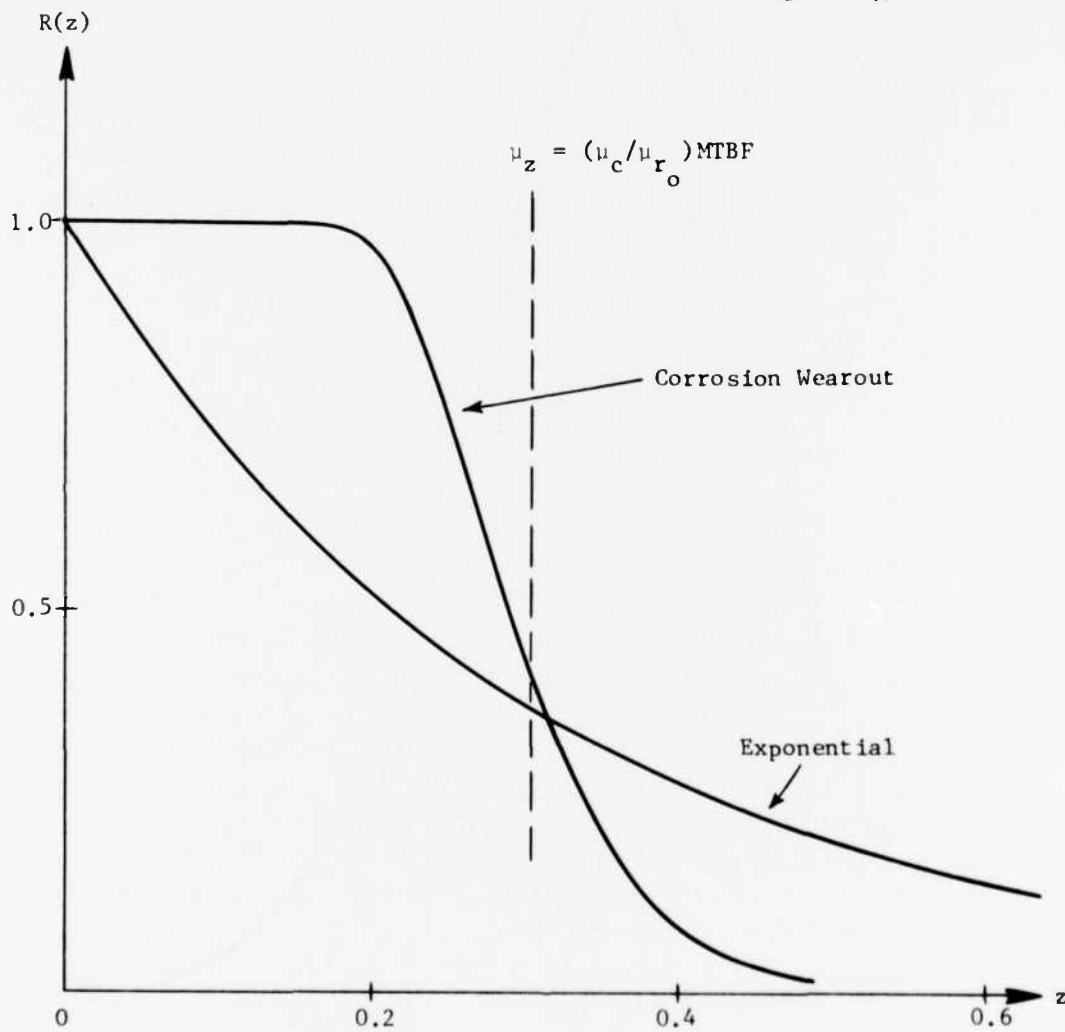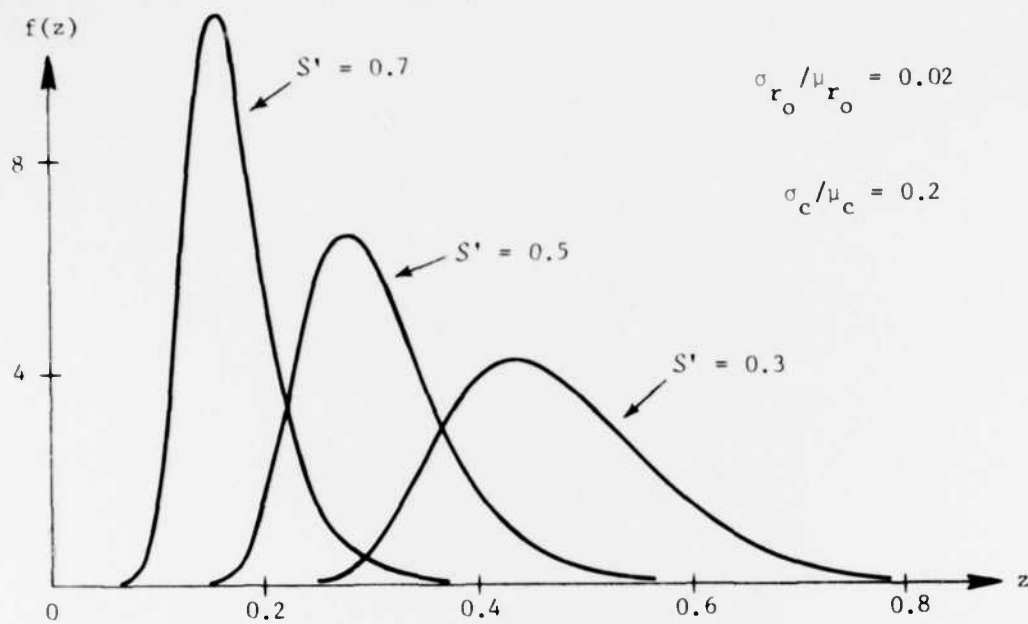
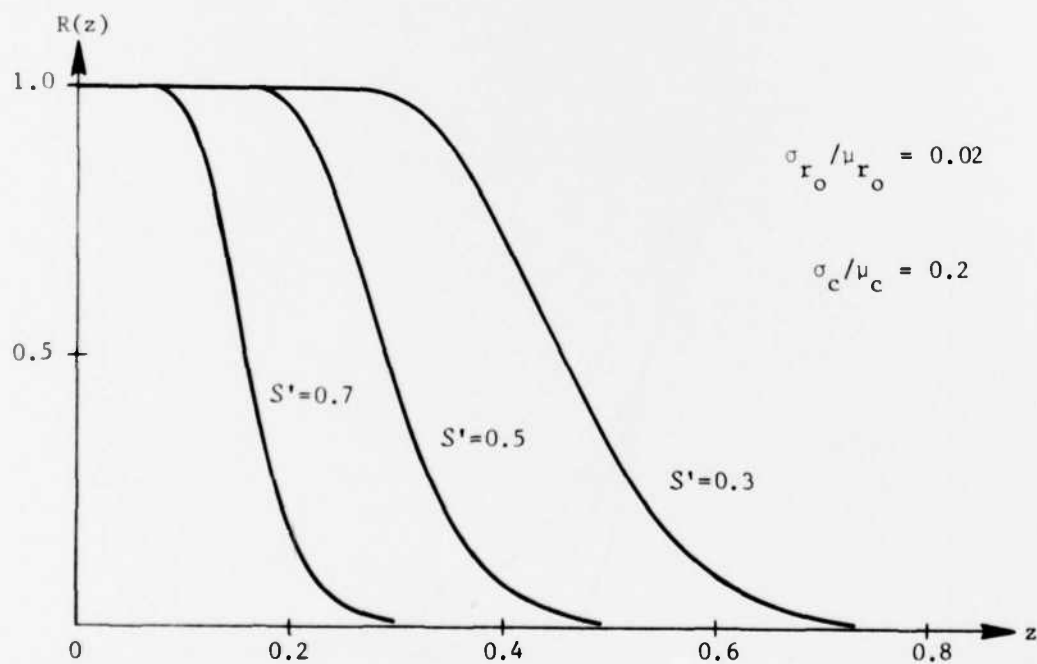$$\sigma_c / \mu_c = 0.2$$

$$S' = 0.5$$



Figure 18. Comparison of Corrosion Wearout and Exponential
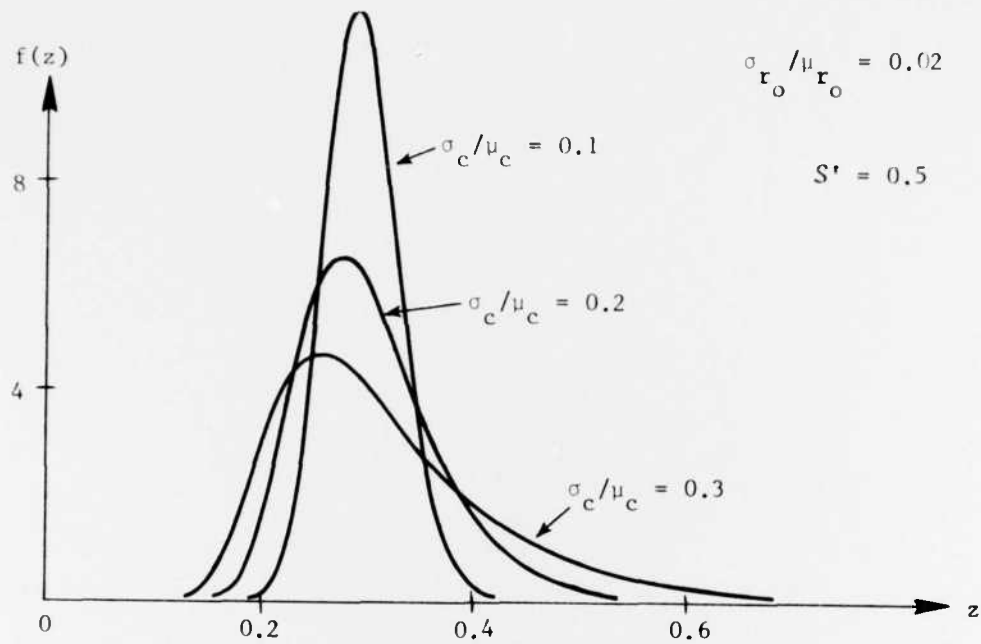Reliability Functions Having the Same MTBF

a. Effect of Endpoint Choice on Time-to-Failure PDF
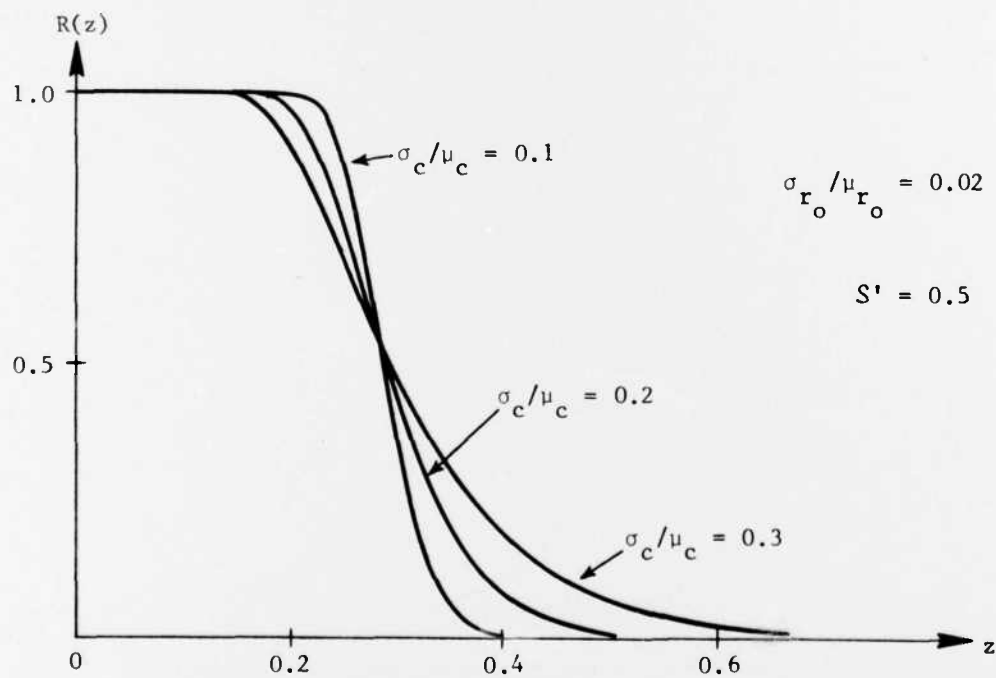


b. Effect of Endpoint Choice on Wearout Reliability

Figure 19. Corrosion Modeling Endpoint Effects

a. Effect of Corrosion Rate Dispersion on Time-to-Failure PDF



b. Effect of Corrosion Rate Dispersion on Wearout Reliability

Figure 20. Corrosion Modeling Dispersion Effects

$$f(\rho_{n,i}) = \frac{n!}{(i-1)!(n-i)!} \rho_{n,i}^{(i-1)}(1 - \rho_{n,i})^{(n-i)}$$
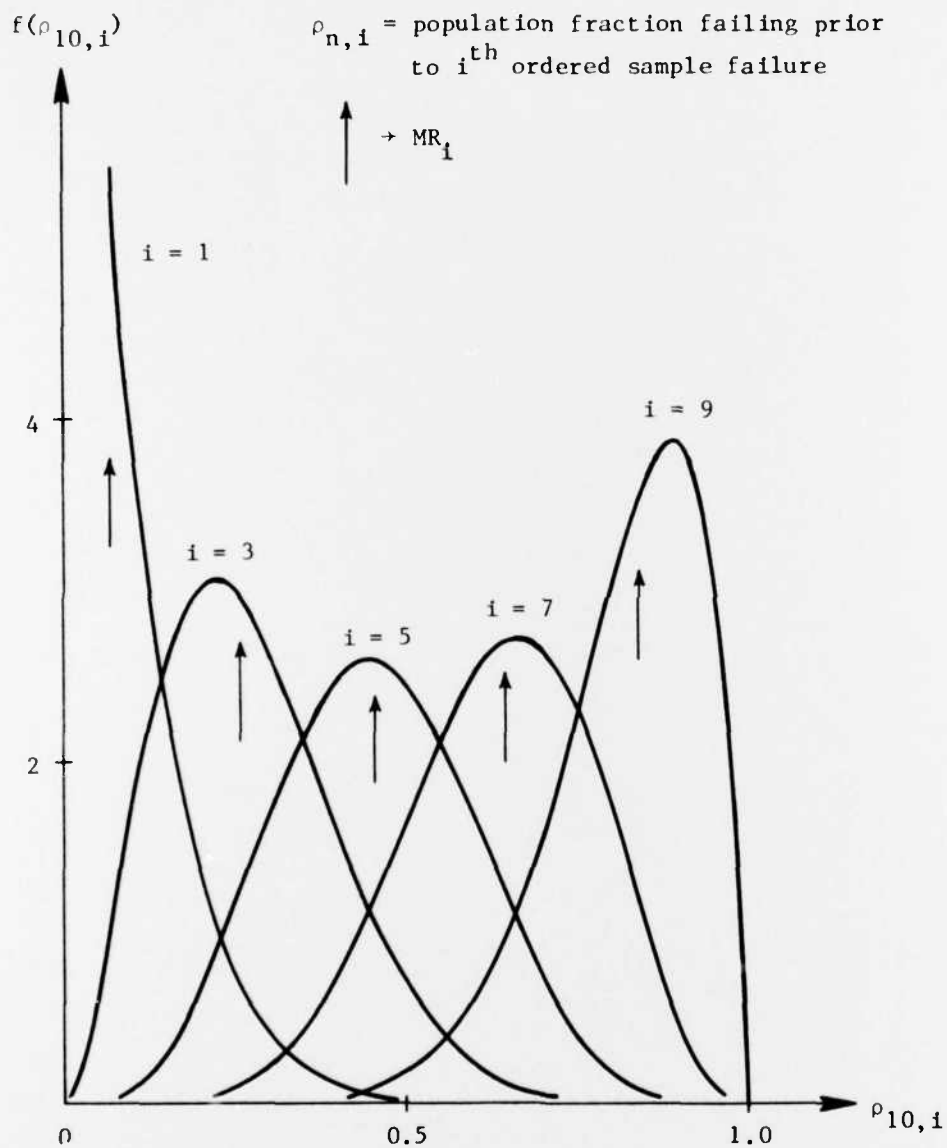
$\rho_{n,i}$ = population fraction failing prior to $i^{th}$ ordered sample failure

$\rightarrow$ MR$_i$
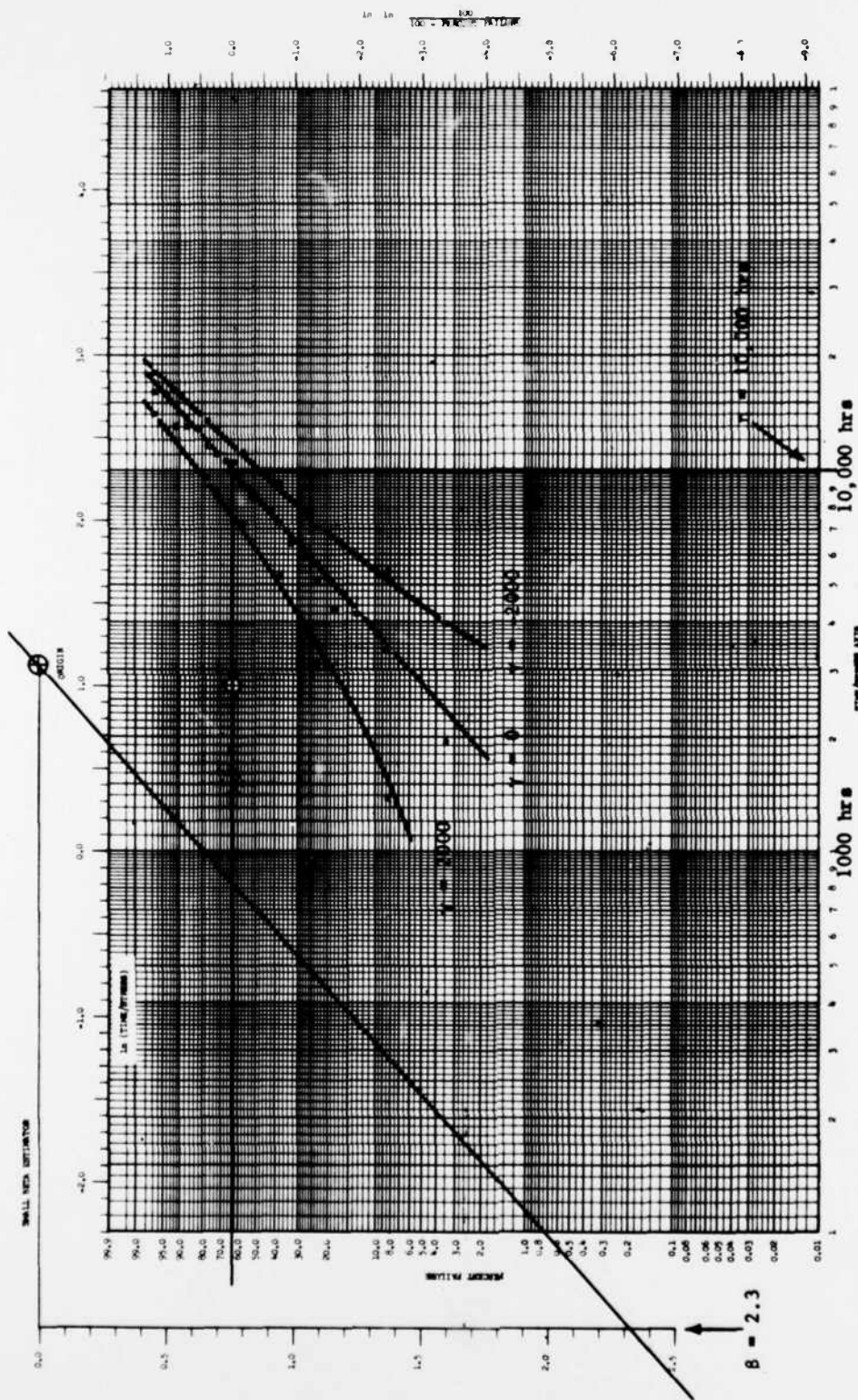


Figure 21. Rank Distributions for Sample Size 10

Figure 22. Simulated Time-to-Failure Data Plotted on Weibull Probability Paper

$P_A$ = Probability of Accepting Lot

$\theta$ = True Equipment MTBF

$\alpha$ = Producer's Risk

$\beta$ = Consumer's Risk

$\theta_L$ = Minimum Acceptable MTBF

$\theta_U$ = Desired MTBF

$k = \theta_U/\theta_L$ = Discrimination Ratio
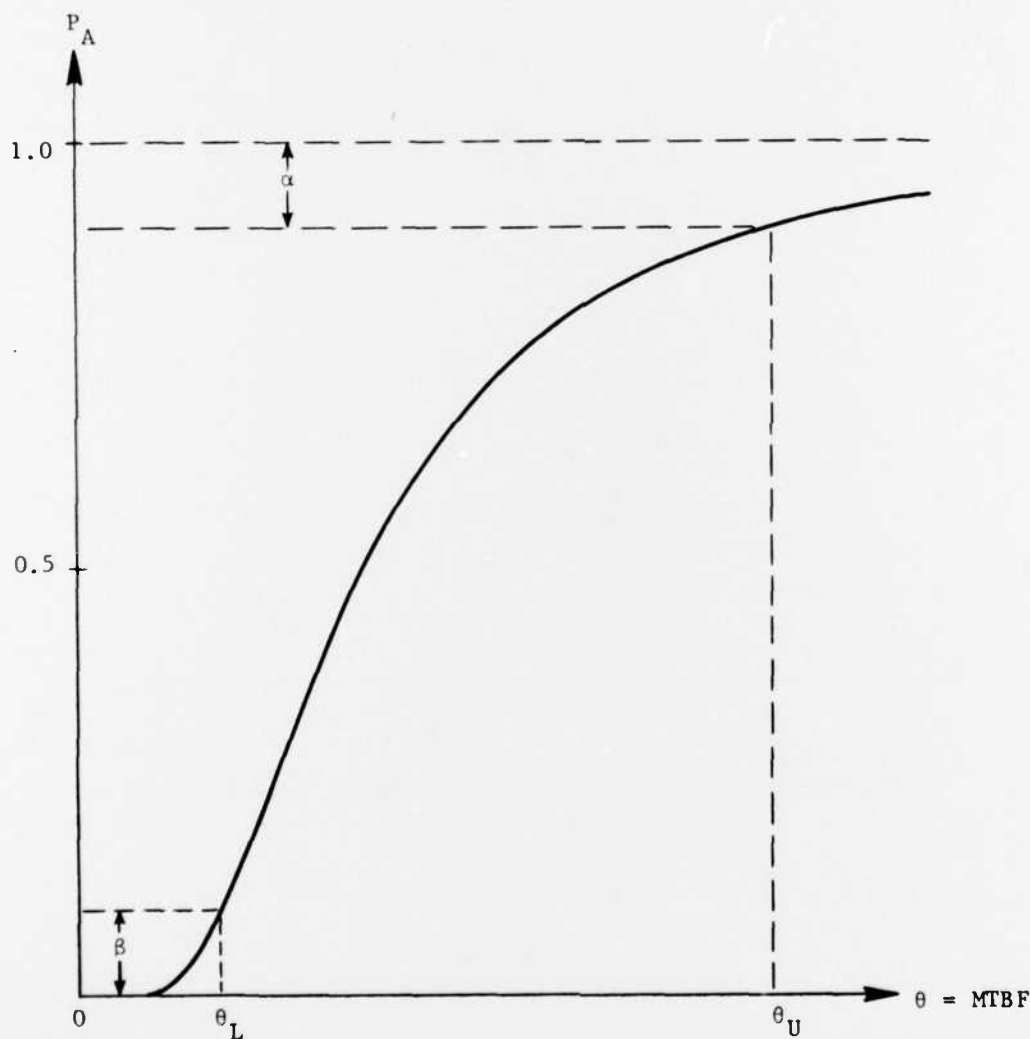


Figure 23. Typical Acceptance Testing Operating Characteristic Curve

97
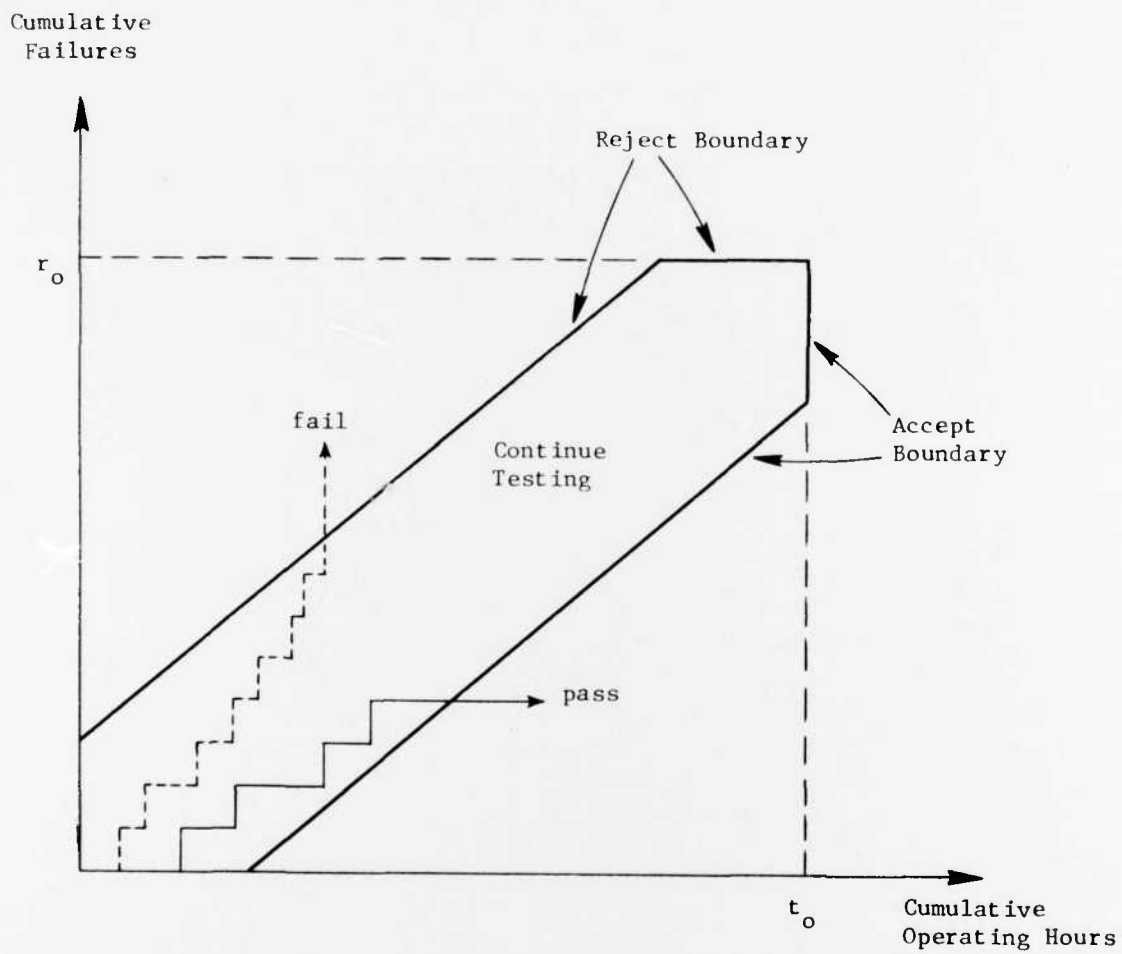
Figure 24. A Typical Sequential Testing Sampling Plan

A: $R = e^{-\mu_\lambda t}$

B: $R = \left[e^{-\mu_\lambda t}\right]\left[e^{(\sigma_\lambda^2 t^2/2)}\right]$

C: Simulated Times to Failure Based on $\lambda$ Normally Distributed with Parameters $\mu_\lambda$ and $\sigma_\lambda = 0.25\mu_\lambda$

Figure 25. Population Reliability of Ten Units-- Deterministic and Distributed Failure Rates Compared

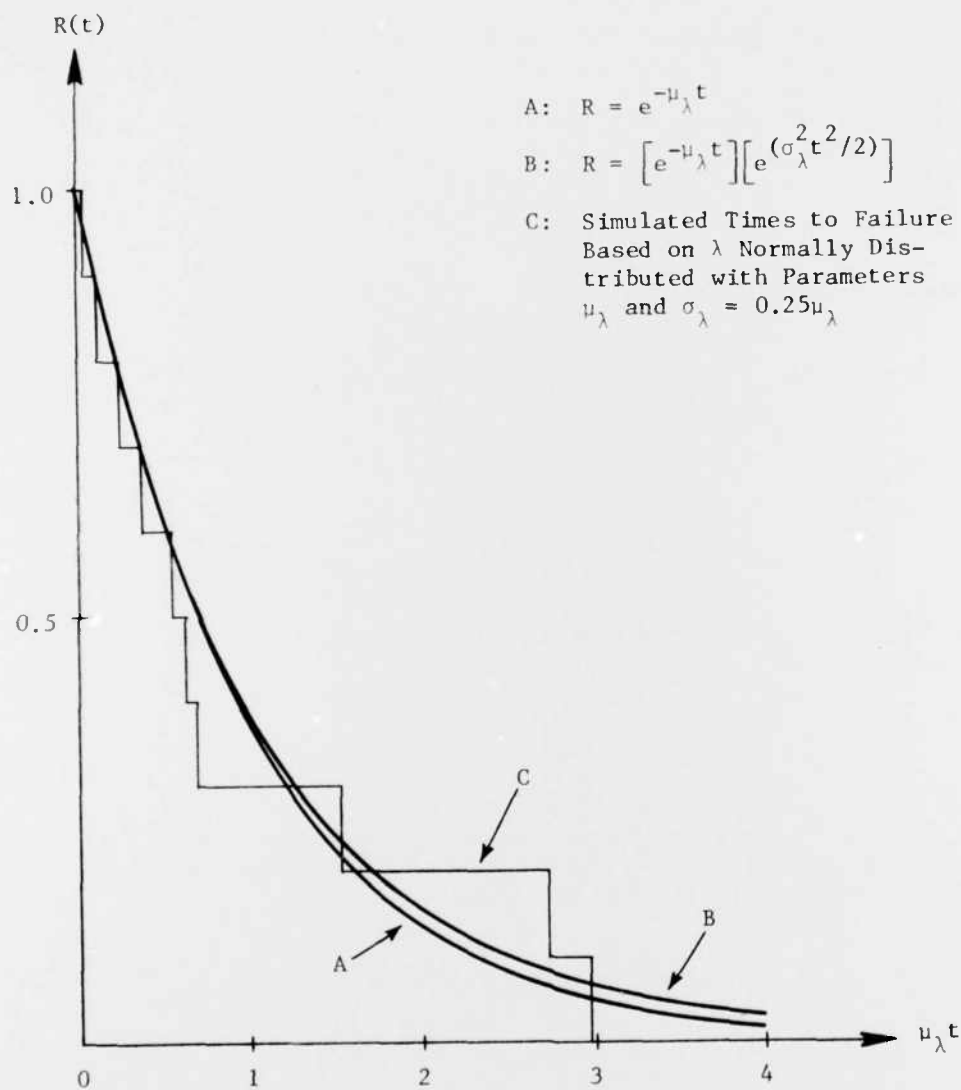## APPENDIX A

### Reliability as a Function of Hazard Rate

Let the hazard rate function (probability of failure per unit time) be represented as $\lambda(t)$. Consider N identical units having hazard rate $\lambda(t)$ to be operational at time t. The probability of a failure occurring in an infinitesimal interval dt at t is $N\lambda(t)dt$ which results in a change $-dN$ in the number of unfailed units remaining. Thus

$$-dN = N\lambda(t)dt . \tag{A1}$$

The variables separate yielding

$$\frac{dN}{N} = -\lambda(t)dt . \tag{A2}$$

Integration yields

$$\ln N = -\int_0^t \lambda(t)dt + \ln N_o , \tag{A3}$$

where $N_o$ is the number of functioning units at t=0. Exponentiation of Eq. (A3) yields

$$\frac{N}{N_o} = \exp\left(-\int_0^t \lambda(t)dt\right). \tag{A4}$$

But $N/N_o$ is the fraction of the initial population that is unfailed, which equals the <u>probability</u> that a single device is unfailed. We identify this with single device reliability R(t). Thus

$$R(t) = \exp\left(-\int_0^t \lambda(t)dt\right). \tag{A5}$$

100

## APPENDIX B

### Transformation of Distributed Random Variables

In this appendix we do not so much derive as simply report results useful to the development of Eqs. (26) and (37) in the body of the text. The interested reader is referred to standard sources such as Ref. 5 for additional information.

Suppose one has specified the probability density function f(x) of a continuous random variable x. If a change of variable

$$y = y(x) \tag{B1}$$

is introduced, then the probability density function of the new variable y is

$$g(y) = f[x(y)] \left| \frac{dx}{dy} \right|. \tag{B2}$$

The sample spaces for x and y are scaled via Eq. (B1). In Eq. (B2) f[x(y)] means that x(y) [the inverse of y(x)] is substituted for x in f(x). The quantity |dx/dy| is the absolute value of the derivative dx/dy which may be more conveniently evaluated as $[dy/dx]^{-1}$. Equation (B2) applies when Eq. (B1) is monotonic over the sample space of x. If this is not the case, then the problem must be decomposed into regions where y(x) is strictly increasing or decreasing and Eq. (B2) applied to each of them separately.

Analogous results can be developed for functions of more than one distributed variable and for discrete distributions. These are not needed for the present purpose.

# APPENDIX C

## Evolution of the Wearout Profile When Failures Are Replaced as They Occur

Imagine that we are dealing with a group of units that exhibit pure wearout behavior with a time-to-failure probability density function $f_1(t)$.
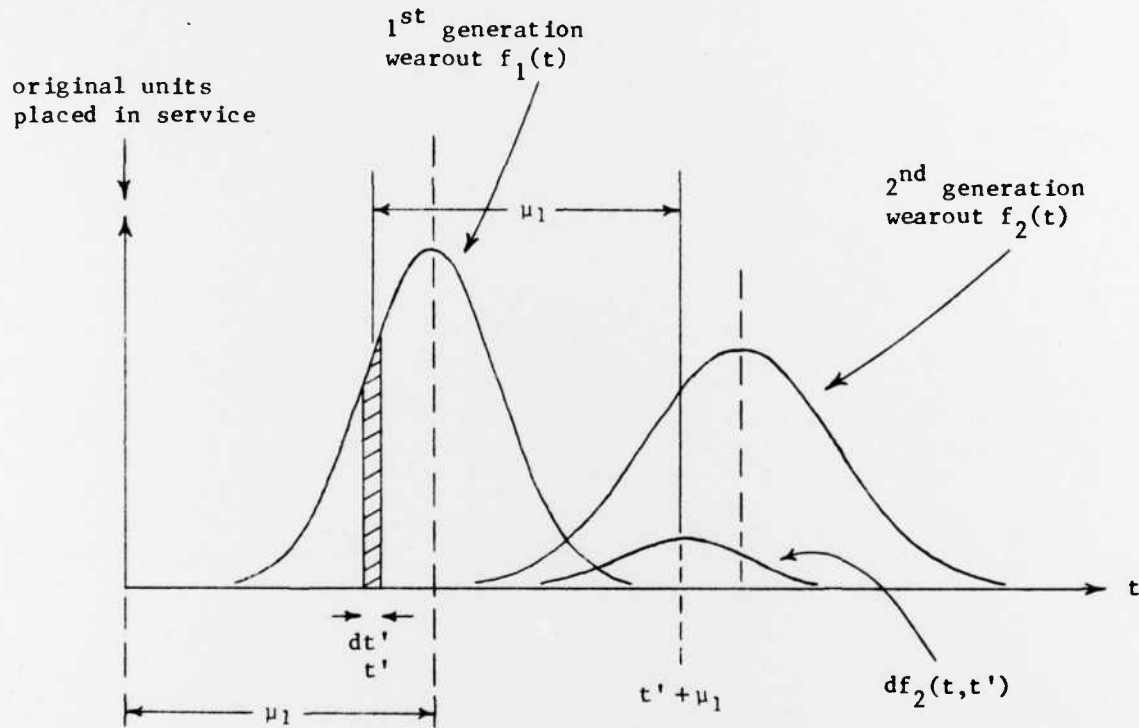


Figure A1.   Graphical Relationship of 1st and 2nd Generation Wearout Profiles

First generation units that wear out and are replaced in the interval dt' at t' contribute a distribution $df_2(t,t')$ to the second generation of times to failure where

$$df_2(t,t') \;=\; \left( \frac{f_1(t'-\mu_1)dt'}{\int f_1(t'-\mu_1)dt'} \right) \left( f_1(t-t'-\mu_1) \right). \tag{C1}$$

Or since $f_1$ is normalized

$$df_2(t,t') \;=\; f_1(t-t'-\mu_1)f_1(t'-\mu_1)dt'. \tag{C2}$$

102

Introducing the variable changes $\alpha = t - 2\mu_1$ and $\beta = t' - \mu_1$ and integrating over t' yields the $2^{nd}$ generation time-to-failure profile

$$f_2(t) = \int_{-\infty}^{\infty} f_1(\alpha - \beta) f_1(\beta) d\beta. \qquad (C3)$$

Equation (C3) is a standard convolution integral. If we take $f_1(t)$ to be a Gauss-normal function, that is

$$f_1(\beta) = \left(\frac{1}{\sqrt{2\pi}\,\sigma_1}\right) \exp\left[-\frac{1}{2}\left(\frac{\beta}{\sigma_1}\right)^2\right] \qquad (C4a)$$

and

$$f_1(\alpha - \beta) = \left(\frac{1}{\sqrt{2\pi}\,\sigma_1}\right) \exp\left[-\frac{1}{2}\left(\frac{\alpha - \beta}{\sigma_1}\right)^2\right], \qquad (C4b)$$

evaluation of Eq. (C3) yields

$$f_2(t) = \left(\frac{1}{\sqrt{2\pi}\,(\sqrt{2}\,\sigma_1)}\right) \exp\left[-\frac{1}{2}\left(\frac{t - 2\mu_1}{\sqrt{2}\,\sigma_1}\right)^2\right]. \qquad (C5)$$

Equation (C5) is itself Gaussian centered at $2\mu_1$ with a standard deviation $\sqrt{2}\,\sigma_1$. This procedure is readily generalized. One finds that the time-to-failure distribution for the $n^{th}$ generation of wearout failures is Gaussian with parameters

$$\mu_n = n\,\mu_1 \qquad (C6a)$$

and

$$\sigma_n = \sqrt{n}\,\sigma_1. \qquad (C6b)$$

For $n = 0$ this description correctly accommodates the simultaneous placement of the original units in service at $t = 0$. (The Gaussian with parameters $\mu_0 = \sigma_0 = 0$ is a Dirac function.) The time-to-failure distributions broaden with increasing generation index number n. Soon overlap effects become dominant and the overall system failure rate obtains via contributions from many generations superimposed.

103

## APPENDIX D

### Binary Synthesis of the Corrosion Model Failure Governing Strength Distribution

Provided the coefficients of variation of the quantities involved are not too large, functions of normally distributed random variables are themselves at least approximately normally distributed. The parameters of the composite distribution may be inferred from the means and standard deviations of the basis variables grouped a pair at a time. This approach is referred to as binary synthesis of normal distributions. Kececioglu discusses the method in Ref. 8 and catalogs the appropriate relationships for a number of elementary operations.

We are interested in evaluating the distribution of residual strengths $S = \pi T (r_o - ct)^2$ for the corrosion wearout model discussed in Section 5.4 of the body of the report. Expressed in the format $x = x[\mu_x, \sigma_x]$ where $\mu_x$ and $\sigma_x$ are the mean and standard deviation of the Gaussian function representing the distribution of the quantity x, our modeling assumptions were:

$$r_o = r_o[\mu_{r_o}, \sigma_{r_o}] \tag{D1a}$$

$$c = c[\mu_c, \sigma_c] \tag{D1b}$$

$$T = T[T, 0] \tag{D1c}$$

$$t = t[t, 0]. \tag{D1d}$$

With this input information we can apply the rules for multiplying a distributed variable by a constant, subtraction of variables, and squaring to obtain the desired results. Thus using the notation $f(x)$ to denote the distribution of x, building the parameters of $f(S)$ proceeds by binary synthesis as follows:

$f(ct):$

$$\mu_{ct} = \mu_c t$$

$$\sigma_{ct} = \sigma_c t$$

$f(r_o - ct):$

$$\mu_{(r_o - ct)} = \mu_{r_o} - \mu_c t$$

$$\sigma_{(r_o - ct)} = \left(\sigma_{r_o}^2 + \sigma_c^2 t^2\right)^{\frac{1}{2}}$$

$f\left((r_o - ct)^2\right):$

$$\mu_{(r_o - ct)^2} = \left(\mu_{r_o} - \mu_c t\right)^2 + \sigma_{r_o}^2 + \sigma_c^2 t^2$$

$$\sigma_{(r_o - ct)^2} = \left[4\left(\mu_{r_o} - \mu_c t\right)^2 \left(\sigma_{r_o}^2 + \sigma_c^2 t^2\right) + 2\left(\sigma_{r_o}^2 + \sigma_c^2 t^2\right)^2\right]^{\frac{1}{2}}$$

104

$f(S):$

$$\mu_S = \pi T \left[ \left( \mu_{r_o} - \mu_c t \right)^2 + \sigma_{r_o}^2 + \sigma_c^2 t^2 \right]$$

$$\sigma_S = \pi T \left[ 4 \left( \mu_{r_o} - \mu_c t \right)^2 \left( \sigma_{r_o}^2 + \sigma_c^2 t^2 \right) + 2 \left( \sigma_{r_o}^2 + \sigma_c^2 t^2 \right)^2 \right]^{\frac{1}{2}}.$$

The quantities $\mu_S = \mu_S(t)$ and $\sigma_S = \sigma_S(t)$ are the time dependent strength distribution parameters required for use in Section 5.4.

105

## FORTRAN IV  Example Curve-Fitting Program

```
C      LEAST SQUARES FIT -- WEIBULL CUMULATIVE PDF
C
C      DESCRIPTION OF PARAMETERS:
C
C              IRUN - DATA SET CATALOG NUMBER
C            NPOINT - NUMBER OF XY DATA PAIRS
C                 X - ARRAY OF VALUES OF INDEPENDENT VARIABLE (TIME, CYCLES, ETC.)
C                 Y - MEDIAN RANKS (ETC.) CORRESPONDING TO THE X'S
C              NPAR - NUMBER OF FITTING PARAMETERS
C              BETA - WEIBULL SHAPE PARAMETER
C               ETA - WEIBULL SCALE PARAMETER
C             GAMMA - WEIBULL LOCATION PARAMETER
C
C      SUBROUTINE REQUIRED: MATINV -- SEE REF. 17 PAGE 302
C
C      DIMENSION AND DP STATEMENTS VALID UP TO NPAR = 6, NPOINT = 100
C
       DIMENSION X(100), Y(100), YTHEOR(100), C(6), DY(6), S(6)
       DOUBLE PRECISION DELSQR, ASCALE(6,6), A(6,6), B(6)
     3 FORMAT(2F10.4)
     4 FORMAT(1H1, 2I5)
     5 FORMAT(1H , 1P6E14.6)
     6 READ(6,4) IRUN, NPOINT
       WRITE(3,4) IRUN, NPOINT
       WRITE(3,5)
       READ(6,3) (X(I), Y(I), I = 1, NPOINT)
       NPAR = 3
C
C          C(I) ARE INITIAL PARAMETER ESTIMATES
C
       READ(6,3) (C(I), I = 1, NPAR)
       DELSAV = 1.E35
    13     DO 16 I = 1, NPAR
       B(I) = 0.
           DO 16 J = 1, NPAR
    16 A(I,J) = 0.
       DELSQR = 0.
             BETA = C(1)
              ETA = C(2)
            GAMMA = C(3)
       BOE = BETA/ETA
```

```fortran
      DO 35 N = 1, NPOINT
      Z = (X(N)-GAMMA)/ETA
      ZEB = Z**BETA
      EXZEB = EXP(-ZEB)
      YTHEOR(N) = 1. - EXZEB
C
C         DY(I) = D(YTHEOR)/D(C(I)) ARE PARTIAL DERIVATIVES
C
      DY(3) = -BOE*ZEB*EXZEB/Z
      DY(2) = Z*DY(3)
      DY(1) = -ALOG(Z)*DY(2)/BOE
      DEL = Y(N) - YTHEOR(N)
      DELSQR = DELSQR + DEL*DEL
         DO 35 I = 1, NPAR
      B(I) = B(I) + DY(I)*DEL
         DO 35 J = 1, NPAR
   35 A(I,J) = A(I,J) + DY(I)*DY(J)
      WRITE(3,5) (C(I), I = 1, NPAR), DELSQR
      IF (DABS(DELSAV-DELSQR) .LT. 0.01*DELSQR) GO TO 48
      IF (DELSAV-DELSQR) 54, 39, 39
   39 DELSAV = DELSQR
C
C     RESCALE CURVATURE MATRIX (DIAGONAL ELEMENTS = 1)
C
         DO 42 I = 1, NPAR
         DO 42 J = 1, NPAR
   42 ASCALE(I,J) = A(I,J)/DSQRT(A(I,I)*A(J,J))
C
C     INVERT MATRIX AND CALCULATE NEW PARAMETERS
C
      CALL MATINV(ASCALE, NPAR, DET)
         DO 46 I = 1, NPAR
         DO 46 J = 1, NPAR
   46 C(I) = C(I) + ASCALE(I,J)*B(J)/DSQRT(A(I,I)*A(J,J))
      GO TO 13
C
C     COMPUTE PARAMETER UNCERTAINTIES
C
   48 RMS = DSQRT(DELSQR/FLOAT(NPOINT-NPAR))
         DO 50 I = 1, NPAR
   50 S(I) = RMS*DSQRT(ASCALE(I,I)/A(I,I))
      WRITE(3,5)
      WRITE(3,5) (S(I), I = 1, NPAR), RMS
      GO TO 6
   54 STOP
      END
```

# END

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